Stochastic Models in Telecommunications for Optimal Design, Control and Performance Evaluation

N. Gautam
Department of Industrial and Manufacturing Engineering,
The Pennsylvania State University,
310 Leonhard Building,
University Park, PA 16802
ngautam@psu.edu

Abstract

In this paper stochastic models for high-speed networks viz. traffic models, buffer content process models, LAN models, etc are considered. The stochastic models are used to obtain end-to-end Quality-of-Service (QoS) measures that the network must guarantee users in the future. The QoS measures or performance measures can be used in optimal design problems and admission control problems. Other aspects such as TCP, routing, leaky buckets, wireless networks, etc are also considered.

1 Introduction

One of the greatest success stories for stochastic models in engineering is in the field of telecommunications. Discrete and fluid queueing models have played a major role in the development of computer and communication networks. There are several branches of telecommunications that use stochastic models, however, in this paper the main focus is in networking systems and other stochastic systems that aide high-performance networking. A few other applications are briefly mentioned later in this introduction.

There are several interesting scenarios in the Internet and the emerging next generation networks (such as Internet2, NGI, etc) where stochastic modeling is applicable. The future networks will carry a wide variety of traffic (Data, Voice, Video, etc) and the users will demand very high-quality from the networks. Therefore it is very important to consider certain performance issues known as Quality-of-Service (QoS). There are four well-known end-to-end QoS measures, viz., loss probability, delay, delay-jitter and bandwidth. They are briefly described as follows:

When messages flow from a source to a destination (end-to-end) through a network, parts of a message or the whole message may be dropped due to unavailable resources (buffer capacity) to store the messages. The probability of delivering a message with some data loss is termed as loss probability. The time between the source sending a message and the destination receiving it is called latency or delay. Typically real-time or multimedia traffic (such as live video conference) can tolerate some loss but have very stringent delay requirements. However data traffic such as emails, fax, file transfers, etc can tolerate some delay but almost zero loss. The other QoS measures are delay-jitter (which is a measure of the variation in the delay) and bandwidth (which is the rate at which messages are processed).

The message flow (will be called traffic henceforth) and the network conditions are extremely stochastic in nature. Given the growth in the Internet as well as users demanding QoS for their applications, it is important to be able to predict the QoS measures as they will have to be guaranteed to the users. Also the QoS measures can be used for **optimal design** and **admission control** of the networks. Some of the main **design** aspects include buffer sizes, link capacities, network parameters, traffic shaping parameters, etc. While exercising **admission control**, the network either rejects an incoming request for connection or accepts it (and provides the required QoS).

As mentioned earlier, the main concentration of this paper will be on high-speed telecommunication networks. Other applications of stochastic processes in communications include coding theory, signal processing, image processing, pattern recognition, speech recognition, etc. Stochastic models using hidden Markov processes (for a thorough exposition of hidden Markov models, see Rabiner [68]), hidden semi-Markov models, Markov decision processes, etc are used in signal processing, image processing, pattern recognition and speech recognition.

Broadly there are three types of telecommunication networks – telephony (telephone networks for voice calls, fax, and also dial-up connections), cable-TV networks (cable, web-TV, etc), and high-speed networks such as the Internet. This paper focuses on high-speed networks with the motivation that in the very near future, internet telephony, video-on-demand, networked homes, multimedia applications, etc will possibly replace telephone and cable-TV networks due to low cost. However, unless the performance of high-speed networks improves greatly this will not be possible!

2 Traffic Models

Traffic flowing through the networks can be classified into several types. Two of the most common traffic types are ethernet packets/frames and ATM cells. Depending on the network segment, all messages are broken down into either packets or cells. The length or size of an ethernet packet ranges anywhere from 60 bytes to 1500 bytes and generally follows a bimodal distribution. The length of ATM cells is fixed at 53 bytes. Therefore the network traffic comprises of millions and billions of these little packets or cells!

One of the most important tasks before evaluating the performance of telecommunication networks is to fit appropriate models for traffic to capture their stochastic nature. Data can be obtained by using "sniffers" on the network and analyzing a "dump" of all the packets or cells that were generated during the time the sniffer was used. The information that can be obtained about each packet or cell by sniffing include: its arrival time, its source, its destination, its length, its type, etc. To fit traffic models, only the time of arrival and packet size are sufficient.

2.1 Hierarchical Networks

Telecommunication networks are typically hierarchical in nature. Generally, traffic can be classified into four levels :

- Application Level : The traffic generated by an application, say, http or telnet or ftp which can vary significantly based on the protocols they follow.
- Source Level : Each workstation or computer can be thought of as a source that generates traffic. This traffic comprises of the traffic generated by different applications that are running on the source. Therefore the traffic that flows on a link that exits the computer is a mixture of the different applications. The process of mixing is known as multiplexing.
- Aggregate Level : Several computer, printers, etc are connected together to form a local area network (LAN). The traffic on a LAN pipe is the aggregated traffic that is multiplexed from all the sources.
- Backbone Level : The LANs are connected together by means of a backbone (say, the Internet backbone), and this forms the Metropolitan Arean Networks (MANs) or the Wide Area Networks (WANs). The traffic on a MAN/WAN pipe is the combination of the traffic from several LANs.

Appropriate traffic models can be used depending on the levels being considered. Some frequently used stochastic models for traffic flow are explained in the next section. Although, different researchers prefer to use different traffic models, the models can be broadly classified into two parts, discrete models and fluid models. In the discrete model each packet or cell is assumed to be a discrete entity that can be of varying sizes. In the fluid models it is assumed that the packets or cells are packed so close to each other that the traffic flow can be assumed to be a fluid flowing across a pipe, maybe at different rates.

2.2 Fluid-flow Traffic Models

In the fluid-flow models it is assumed that traffic is in the form of fluid which flows through a pipe at different rates at different times. For example, fluid flows at rate $r(1)$ bytes per second for a random amount of time t_1, then flows at rate $r(2)$ bytes per second for a random amount of time t_2, and so on. This behaviour can be captured as a discrete stochastic process that jumps from one state to another whenever the traffic flow rate changes. This can be formalized as a stochastic process $\{Z(t), t \geq 0\}$ that is in state $Z(t)$ at time t. Fluid flows in the pipe at rate $r(Z(t))$ at time t. Researchers have used different models for the $\{Z(t), t \geq 0\}$ process that are summarized in the following description :

2.2.1 DTMC environmental processes

Consider a DTMC where a transition occurs every θ seconds. When the DTMC is in state i, $\theta r(i)$ bytes flow through the pipe. Let $P = [p_{ij}]$ be the transition probability matrix, where p_{ij} is the probability that the DTMC goes from state i to state j in one-step. For an irreducible, aperiodic and positive recurrent DTMC, let π be the steady-state distribution such that $\pi = \pi P$ and $\pi \mathbf{a} = 1$ with $\mathbf{a}$ being a column vector of ones.

2.2.2 CTMC environmental processes

Let $\{Z(t), t \geq 0\}$ be an irreducible, finite state CTMC with generator matrix Q. When the CTMC is in state i, traffic flows at rate $r(i)$. Let $R = \text{diag}[r_{ii}]$, where $r_{ii} = r(i)$. Let p be the stationary distribution of the CTMC such that $pQ = 0$ and $p\mathbf{a} = 1$ with $\mathbf{a}$ being a column vector of ones.

2.2.3 Alternating Renewal environmental processes

This is sometimes known as the on-off traffic where traffic either flows at the maximum link (or pipe) capacity r bytes per second or no traffic flows. The up times or on times (when traffic flows through the pipe) are distributed according to a general CDF $U(\cdot)$. The down or off times (when traffic does not flow through the pipe) are distributed according to a general CDF $D(\cdot)$.

2.2.4 SMP environmental processes

Consider a Semi-Markov Process (SMP) $\{Z(t), t \geq 0\}$ on state space $\{1, 2, ..., \ell\}$. Fluid is generated at rate $r(i)$ at time t when the SMP is in state $Z(t) = i$. Let S_n denote the time of the nth jump epoch in the SMP with $S_0 = 0$. Define Z_n as the state of the SMP immediately after the nth jump, i.e.,

$$Z_n = Z(S_n+).$$

Let

$$G_{ij}(x) = P\{S_1 \leq x; Z_1 = j | Z_0 = i\}. \tag{1}$$

The kernel of the SMP is

$$G(x) = [G_{ij}(x)]_{i,j=1,\ldots,\ell} \ .$$

Note that $\{Z_n, n \geq 0\}$ is a discrete time Markov chain (DTMC) which is embedded in the SMP. Assume that this DTMC is irreducible and recurrent with transition probability matrix

$$P = G(\infty).$$

Let

$$G_i(x) = P\{S_1 \leq x | Z_0 = i) = \sum_{j=1}^{\ell} G_{ij}(x)$$

and the expected time the SMP spends in state i be

$$\tau_i = E(S_1 | Z_0 = i).$$

Let

$$\pi_i = \lim_{n \to \infty} P\{Z_n = i\}$$

be the stationary distribution of the DTMC $\{Z_n, n \geq 0\}$. It is given by the unique non-negative solution to

$$\pi = \pi P \text{ and } \sum_i \pi_i = 1.$$

The stationary distribution of the SMP is given by

$$p_i = \lim_{t \to \infty} P\{Z(t) = i\} = \frac{\pi_i \tau_i}{\sum_{m=1}^{\ell} \pi_m \tau_m}. \tag{2}$$

2.2.5 MRGP environmental processes

Consider a regular Markov regenerative process $\{Z(t), t \geq 0\}$. For the definition of MRGP see Cinlar [20], who calls it a semi-regenerative process. Also see Heyman and Sobel [42] and Kulkarni [51]. Let $\{(Y_n, S_n), n \geq 0\}$ be an embedded Markov renewal sequence in the MRGP. Assume that $\{Y_n, n \geq 0\}$ has a finite state-space $\{1, 2, ..., \ell\}$. From the definition of the Markov renewal sequences,

$$P\{Y_{n+1} = j, S_{n+1} - S_n \leq x | Y_n = i, S_n, ..., Y_0, S_0\} = P\{Y_1 = j, S_1 \leq x | Y_0 = i\} \tag{3}$$

for all $x \geq 0$, and $i, j = 1, 2, ..., \ell$. Furthermore, given the history $\{Z(t), 0 \leq t \leq S_n\}$ and $\{(Y_k, S_k), 0 \leq k \leq n\}$, the future of the $Z(t)$ process, viz., $\{Z(t), t \geq S_n\}$, depends on the past only via Y_n. Assume that the MRGP is regular, i.e., $S_n \to \infty$ with probability 1 as $n \to \infty$. It is clear from Eq. (3) that $\{Y_n, n \geq 0\}$ is a discrete time Markov chain

(DTMC). Assume it is irreducible. Let $\mu_k = E[S_1|Y_0 = k]$, $\pi = (\pi_k)$ is a positive solution to $\pi = \pi G(\infty)$, and,

$$\alpha_{kj} = E(\text{ time spent by the } Z(t) \text{ process in state } j \text{ during } [0, S_1)|Y_0 = k).$$

Then,

$$p_j = \lim_{t\to\infty} P\{Z(t) = j\} = \frac{\sum_k \pi_k \alpha_{kj}}{\sum_k \pi_k \mu_k}.$$

2.3 Discrete Traffic Models

In the discrete traffic models it is assumed that the packet flow is in the form of discrete entities (like cars going on a highway) through the pipes. Therefore it is important to characterize the arrival process and the size of the packets. Some typical arrival processes, frequently used by researchers are explained below. The packet size distribution is usually a bimodal empirical distribution and is not discussed here.

2.3.1 Poisson Process

If the interarrival distribution of the packets can be modeled using an independent and identically distributed exponential distribution, then the packet arrival process can be characterized as a Poisson process with parameter λ, where λ is the mean arrival rate (in terms of number of packets per unit time).

2.3.2 MMPP (Markov-Modulated Poisson Process)

Whenever a new connection is established through a pipe or an existing connection is terminated, the mean packet flow rate (number of packets per unit time) increases or decreases respectively. To account for this behavior, the traffic arrival process is modeled as an MMPP. Consider a CTMC $\{Y(t), t \geq 0\}$ with generator matrix Q. When the CTMC is in state i, packets flow according to a Poisson process with mean rate $\lambda(i)$. This captures the change in mean arrival rates effectively.

2.3.3 BMAP (Batch Markovian Arrival Process)

The batch Markovian arrival process (BMAP) is explained in detail in Lucantoni [58]. The BMAP is a generalization of the Markovian arrival process (MAP) which was introduced by Lucantoni et al [57]. A special case of the MAP are the phase type renewal processes and the MMPP. The following definition and properties of BMAP is reproduced from Lucantoni et al [59].

Consider a series of $m \times m$ matrices D_k, $k \geq 0$, such that D_0 has negative diagonal elements and nonnegative off-diagonal elements and for $k \geq 1$, D_k are nonnegative. Define an irreducible infinitesimal generator D such that

$$D = \sum_{k=0}^{\infty} D_k.$$

To assure that arrivals will occur assume that $D \neq D_0$.

Consider a two-dimensional Markov process $\{N(t), J(t), t \geq 0\}$ on the state space $\{(i,j) : i \geq 0, 1 \leq j \leq m\}$ with an infinitesimal generator Q given by

$$Q = \begin{bmatrix} D_0 & D_1 & D_2 & D_3 & \dots \\ & D_0 & D_1 & D_2 & \dots \\ & & D_0 & D_1 & \dots \\ & & & D_0 & \dots \\ & & & & \dots \end{bmatrix}$$

Here, $N(t)$ counts the number of attivals in time t and $J(t)$ represents a state or phase. For example, a transition from state (i,j) to state $(i+k,l)$, $k \geq 1$, $1 \leq j,l \leq m$ denotes a batch arrival of size k and thus the batch size can depend on j and l. The matrix D_0 is nonsingular and the sojourn time in the set of states $\{(i,j) : 1 \leq j \leq m\}$ is finite w.p. 1. Thus the arrival process does not terminate.

Let π denote the stationary probability vector of the Markov process with generator D such that

$$\pi D = 0, \qquad \pi e = 1.$$

The mean arrival rate of the process is hence

$$\lambda = \pi \sum_{k=1}^{\infty} k D_k e = \pi d.$$

One can think of D_0 as governing transitions in the phase process which do not generate arrivals and D_k as the rate of arrivals of size k (with the appropriate phase change). As a simple example, for Poisson arrivals with mean arrival rate λ, $m = 1$, $D_0 = -\lambda$, $D_1 = \lambda$ and $D_k = 0$ for all $k \geq 2$.

2.3.4 Fractals or Self-similar Arrival Process

Willinger et all [79] describe the latest developments and advances in using self-similar traffic for performance modeling of high-speed telecommunication networks. Here, the notations and descriptions follow Willinger and Paxson [78].

Experimental traces of traffic processes exhibit high spatial variability and long-range dependence (autocorrelations with a power law decay). Heavy-tailed distributions (such as Pareto distributions) with infinite variance are used to model the extreme spacial variability. Typical probability distributions $[F(\cdot)]$ are of the form

$$1 - F(x) = \kappa_1 x^{-\beta},$$

where κ_1 is a positive (finite) constant independent of x and the tail index β is such that $0 < \beta < 2$. A fractional Gaussian noise is used to model the fractal or long-range dependent or self-similar behavior. A covariance-stationary Gaussian process $X = (X_k : k \geq 1)$ is called a fractional Gaussian noise with Hurst parameter $H \in [0.5, 1)$ if the autocorrelation between X_n and X_{n+k}, $k \geq 0$, is given by

$$cor(X_n, X_{n+k}) = 0.5 \left\{ (k+1)^{2H} - 2(k)^{2H} + (k-1)^{2H} \right\}.$$

The Hurst parameter H quantifies the strength of the fractal scaling.

A discrete-time, covariance-stationary, zero-mean stochastic process $X = (X_k : k \geq 1)$ is called exactly self-similar or fractal with scaling parameter $H \in [0.5, 1)$ if for all levels of aggregation (or resolution), $m \geq 1$,

$$X^{(m)} = m^{H-1} X,$$

where the aggregated processes $X^{(m)}$ are defined by

$$X^{(m)}(k) = \frac{X_{(m-1)k+1} + \ldots + X_{km}}{m}, \quad k \geq 1.$$

For an exactly self-similar process with scaling parameter H,

$$Var X^{(m)} = \kappa_1 m^{2H-2}.$$

2.3.5 Fractional Brownian Motion vs. Levy Processes

The following is directly adapted from Konstantopoulos and Lin [48]. A Levy motion $\{Z_t\}$ is a process with stationary independent increments, and its marginal distribution is a stable random variable (invariant under affine transformations). It is a self-similar process with Hurst constant $1/\alpha$: $\{Z_{tx}, t \geq 0\} \stackrel{\mathcal{D}}{=} \{x^{1/\alpha} Z_t, t \geq 0\}$. Here, $\{Z_t\}$ is a zero mean process and Z_1 has finite β-moments for any $0 < \beta < \alpha$. Since $\alpha > 1$, $\{Z_t\}$ is a martingale, in fact an

L^β martingale for any $1 < \beta < \alpha$. Note that the fractional Brownian motion is not even a semi-martingale. Konstantopoulos and Lin [48] consider multiplexing of multiple number of sessions such that the sessions arrive according to a Poisson process with mean rate λ and each session remains active for a random time T with a heavy-tailed distribution

$$1 - F(x) = P\{T > x\} \sim \kappa x^{-\alpha}, \quad \text{as } x \to \infty$$

for a fixed $1 < \alpha < 2$. Also, when each session is active, traffic is generated at the rate of 1 per unit time. Konstantopoulos and Lin [48] show that (i) in the limit the arrival processes converge to a Levy motion with stable non-Gaussian independent increments, and, (ii) the autocorrelation function is asymptotically that of a fractional Brownian motion and not a Levy motion!

3 Network Performance Using Traffic Models

When an application (also called source) sends a message to a destination, the message traverses several nodes (also called hops or network interfaces) before reaching the destination. The messages are stored in buffers at the nodes briefly before proceeding on to the next hop. At these nodes, traffic from other applications are either superposed along with or split from this application's traffic. In this section the traffic models considered in Section 2 will be used to model buffer content processes and evaluate performance measures.

3.1 Single Class Single Buffer Content Processes

To begin the analysis, first concentrate on the simplest model: a network with a single node, a single class of traffic, hence a single buffer. The main aim is to obtain the probability distribution of the buffer contents in the long-run given a traffic model of the input to the buffer and the buffer emptying scheme.

3.1.1 Effective bandwidths of all the traffic model types

First consider the source generating traffic into the buffer. The concept of effective bandwidth of traffic generated by a source or traffic stream flowing through a telecommunication pipe or link is explained. Let $A(t)$ be the total amount of traffic (fluid or discrete) generated by a source or flowing through a pipe over time $(0, t]$. For the following analysis consider a fluid model. Note that it is straightforward to perform similar analysis for discrete models as well.

Consider a stochastic process $\{Z(t), t \geq 0\}$ that models the traffic flow. Also let $r(Z(t))$ be the rate at which the traffic flows at time t. Then

$$A(t) = \int_0^t r(Z(u))du. \tag{4}$$

The *asymptotic log moment generating function* (ALMGF) of the traffic is defined as

$$h(v) = \lim_{t\to\infty} \frac{1}{t} \log E\{\exp(vA(t))\}. \tag{5}$$

Using Equation (5) one can show that $h(v)$ is an increasing, convex function of v and for all $v > 0$,

$$r^{mean} \leq h'(v) \leq r^{peak}. \tag{6}$$

where

$$r^{mean} = E(r(Z(\infty))),$$

$$r^{peak} = \sup_z \{r(z)\},$$

and $h'(v)$ denotes the derivative of $h(v)$ with respect to v.

The *Effective Bandwidth* of the traffic is defined as

$$eb(v) = h(v)/v. \tag{7}$$

It can be shown that $eb(v)$ is an increasing function of v and

$$r^{mean} \leq eb(v) \leq r^{peak},$$

and,

$$\lim_{v\to 0} eb(v) = r^{mean} \quad \text{and} \quad \lim_{v\to\infty} eb(v) = r^{peak}.$$

It is not easy to calculate the effective bandwidths using Equation (7). However, when $\{Z(t), t \geq 0\}$ is a Continuous Time Markov Chain (CTMC), or a regenerative process, or a Markov Regenerative Process (MRGP), one can compute the effective bandwidths more easily. The methods are illustrated briefly.

1. **CTMC Source :** Elwalid and Mitra [27], and Kesidis et al [47], use eigenvalue techniques to show how to compute the effective bandwidths of sources that are modulated by CTMCs as follows. Let $\{Z(t), t \geq 0\}$ be an irreducible, finite state CTMC with generator matrix Q. When the CTMC is in state i, the source generates fluid at rate

$r(i)$. Let $R = \text{diag}[r_{ii}]$, where $r_{ii} = r(i)$. Let $e(M)$ denote the largest real-eigenvalue of a square matrix M. Then,

$$h(v) = e(Q + vR). \tag{8}$$

2. **MRGP Source :** Kulkarni [50] and Gautam [33] show how to compute the effective bandwidths of sources that are modulated by Markov Regenerative Processes and Regenerative Processes. Let $\{Z(t), t \geq 0\}$ be an m-state Markov Regenerative Process (MRGP). Let $\{(Y_n, S_n), n \geq 0\}$ be an embedded Markov renewal sequence in the MRGP. Assume that $\{Y_n, n \geq 0\}$ is an irreducible Discrete Time Markov Chain (DTMC) with a finite state-space $\{1, 2, \ldots, m\}$. Let

$$F_1 = \int_0^{S_1} r(Z(t))dt$$

be the total fluid generated by the source during $[0, S_1]$. Define

$$\Lambda_{ij}(u, v) = E\{e^{-uS_1+vF_1}; Y_1 = j \mid Y_0 = i\}, \tag{9}$$

for $i, j = 1, 2, \ldots, m$ and $-\infty < u, v < \infty$. Let

$$\Lambda(u, v) = [\Lambda_{ij}(u, v)]$$

be an $m \times m$ matrix. Let $e(\Lambda(u, v))$ be the largest real-positive eigenvalue of $\Lambda(u, v)$.

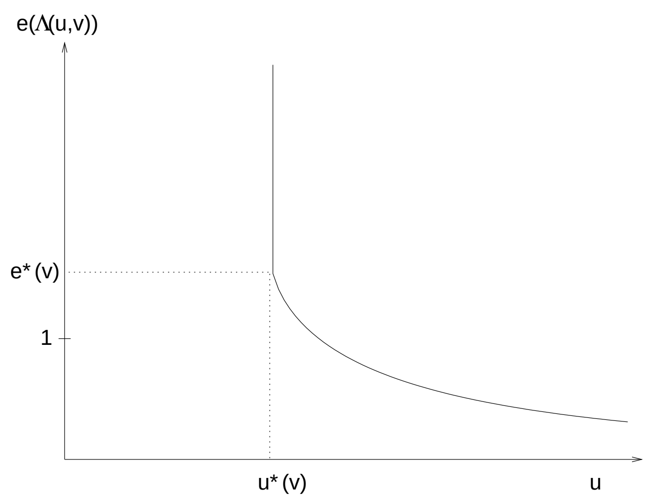

Figure 1: $e(\Lambda(u, v))$ vs u

Define

$$e^*(v) = \sup_{\{u>0:e(\Lambda(u,v))<\infty\}} \{e(\Lambda(u, v))\}$$

and

$$u^*(v) = \inf\{u > 0 : e(\Lambda(u, v)) < \infty\}.$$

Then for a given v,

(a) if $e^*(v) \geq 1$ (see Figure 1), $h(v)$ is a unique solution to $e(\Lambda(h(v), v)) = 1$,

(b) if $e^*(v) < 1$ (see Figure 2), $h(v) = u^*(v)$.

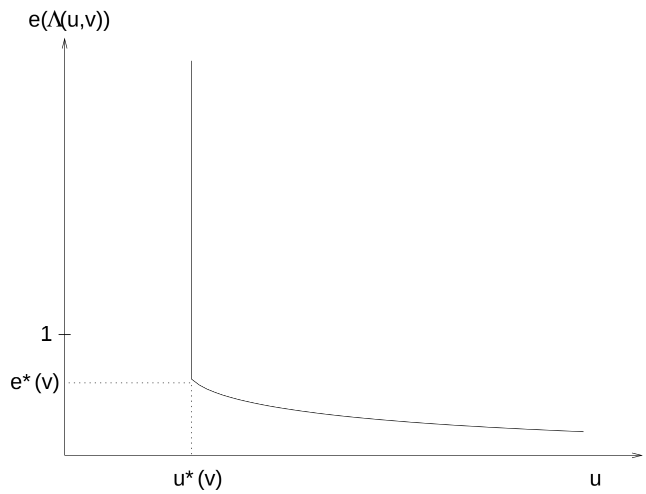

Figure 2: $e(\Lambda(u, v))$ vs u

3. **Regenerative Source :** Assume that $\{Z(t), t \geq 0\}$ is a regenerative process with regeneration epochs $\{S_n, n \geq 0\}$, with $S_0 = 0$. Let

$$F_1 = \int_0^{S_1} r(Z(t))dt$$

be the total fluid generated by the source during $[0, S_1]$. Define

$$\Lambda(u, v) = E\{e^{-uS_1 + vF_1}\}. \quad (10)$$

Since $\Lambda(u, v)$ is a scalar, $e(\Lambda(u, v)) = \Lambda(u, v)$. Following the technique in the MRGP case above, define

$$e^*(v) = \sup_{\{u>0: e(\Lambda(u,v))<\infty\}} \{e(\Lambda(u, v))\}$$

and

$$u^*(v) = \inf\{u > 0 : e(\Lambda(u, v)) < \infty\}.$$

For a given v,

(a) if $e^*(v) \geq 1$, $h(v)$ is a unique solution to $e(\Lambda(h(v), v)) = 1$,

(b) if $e^*(v) < 1$, $h(v) = u^*(v)$.

Table 1 summarizes the effective bandwidths of some discrete ATM traffic in the forms of cells (see Krishnan et al [49] for the calculation of effective bandwidths for traffic modeled by fractional brownian motion).

SOURCE TYPE	**EFFECTIVE BANDWIDTH**
constant arrival rate of R cells/second	R
Poisson source with intensity R cells/second	$\frac{R(e^{\delta}-1)}{\delta}$
irreducible and aperiodic *discrete-time Markov source* with transition probability matrix $\mathbf{P}$, rate matrix $\psi = diag(\psi_1, \ldots, \psi_m)$ where m is the size of the state space of of the DTMC, ψ_i is the number of cells that arrive when in state i, and $\rho(\mathbf{A})$ is the spectral radius of matrix $\mathbf{A}$	$\frac{R \log[\rho(e^{\delta\psi}\mathbf{P})]}{\delta}$
Markov Modulated Poisson Process (MMPP) with intensity $\psi = diag(\psi_i)$, where ψ is a function of a CTMC with infinitesimal generator $\mathbf{Q}$, and $\mu(\mathbf{A})$ is the largest eigenvalue of the matrix $\mathbf{A}$	$\frac{\mu(\mathbf{Q}+(e^{\delta}-1)\psi)}{\delta}$

Table 1: Effective Bandwidth of Input Sources.

3.1.2 Approximate Methods And Bounds (via martingales)

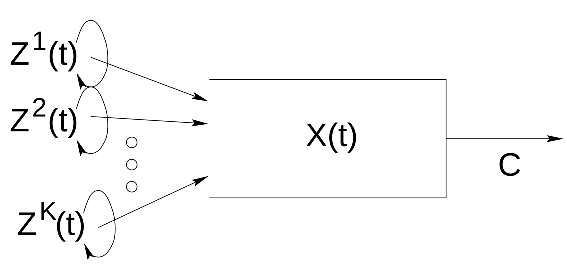

Figure 3: Single Buffer Fluid Model

Consider a single buffer that admits a single-class traffic from K independent sources, each driven by a random environment process $\{Z^k(t), t \geq 0\}$ (see Figure 3). Note that $Z^k(t)$ can be thought of as the state of the kth input source ($k = 1, 2, \ldots, K$) at time t. When source k is in state $Z^k(t)$, it generates fluid at rate $r^k(Z^k(t))$ into the buffer. Let $X(t)$ be the amount of fluid in the buffer at time t. The buffer has infinite capacity and is serviced by a channel of constant rate c. The dynamics of the buffer-content process $\{X(t), t \geq 0\}$ is described by

$$\frac{dX(t)}{dt} = \begin{cases} \sum_{k=1}^{K} r^k(Z^k(t)) - c & \text{if } X(t) > 0 \\ \{\sum_{k=1}^{K} r^k(Z^k(t)) - c\}^+ & \text{if } X(t) = 0. \end{cases} \tag{11}$$

where $\{x\}^+ = \max(x, 0)$. The solution is given by (see Kulkarni and Rolski [54])

$$X(t) = \sup_{0 \leq u \leq t} \left(Y(t), \int_u^t \left(\sum_{k=1}^{K} r^k(Z^k(s)) - c \right) ds \right),$$

where

$$Y(t) = X(0) + \int_0^t \left(\sum_{k=1}^{K} r^k(Z^k(s)) - c \right) ds.$$

It has been shown in Kulkarni and Rolski [54] that the buffer-content process $\{X(t), t \geq 0\}$ is stable if

$$\sum_{k=1}^{K} E\{r^k(Z^k(\infty))\} < c, \tag{12}$$

in which case $X(t) \to X$ in distribution with

$$X = \sup_{u \leq 0} \int_u^0 \left(\sum_{k=1}^{K} r^k(Z^k(s)) - c \right) ds. \tag{13}$$

Exact expressions for the buffer content distribution $P\{X > x\}$ can be obtained only for special environment processes like CTMCs.

Let $h_k(v)$ and $eb_k(v)$ be the ALMGF and effective bandwidths of source k respectively. In Kesidis et al [47], it is shown that

$$\lim_{B \to \infty} P(X > B) e^{B\theta} \to \omega, \tag{14}$$

for some positive finite constant ω, where θ is the solution to

$$\sum_{k=1}^{K} \frac{h_k(\theta)}{\theta} = c. \tag{15}$$

Therefore the effective bandwidth approximation states that one can show that in the long-run, the buffer content distribution $P(X > B) \approx e^{-B\theta}$ (neglect the effect of ω by setting it to 1).

CDE Approximation

It can be shown that the effective-bandwidth approximation is very conservative for most engineering applications, mainly because the statistical multiplexing gains are not taken advantage of. In this subsection the use of Chernoff Dominant Eigenvalue (CDE) approximation (see Elwalid et al [30] and [28]) to further fine tune the effective-bandwidth analysis is explained.

Consider the model in Figure 3. The CDE approximation for the tail probability is given by

$$P\{X > x\} = \lim_{t\to\infty} P\{X(t) > B\} \approx L\, e^{-\theta B} \tag{16}$$

where L is the fraction of the fluid that would be lost if there was no buffer and θ is as in Equation (15). Note that L is an estimate of ω in Equation (14).

Mathematically, L can be written as

$$L = \lim_{t\to\infty} \frac{\int_0^t \left\{ \left[\sum_{k=1}^K r^k(Z^k(t)) \right] - c \right\}^+ dt}{\int_0^t \left\{ \sum_{k=1}^K r^k(Z^k(t)) \right\} dt}. \tag{17}$$

Note that L is a function of c and the parameters of each of the K sources. Typically it may not be computationally simple to calculate L exactly in many applications. Hence Elwalid et al [30] suggest a method of estimating L by using Chernoff's theorem.

The input sources are characterized by a function $m_k(w)$, which is similar to the ALMGF ($h_k(v)$), and is defined as

$$m_k(w) = \lim_{t\to\infty} \log E\{\exp(w r^k(Z^k(t)))\}. \tag{18}$$

Let

$$s^* = \sup_{w\ge 0} \left\{ c\,w - \sum_{k=1}^K m_k(w) \right\}.$$

and w^* be obtained by solving

$$\sum_{k=1}^K m_k'(w^*) = c.$$

Then the Chernoff estimate of L as given in Elwalid et al [30] and [28] is

$$L \approx \frac{\exp(-s^*)}{w^* \sigma(w^*) \sqrt{2\pi}}, \tag{19}$$

where

$$\sigma^2(w^*) = \sum_{k=1}^K m_k''(w^*).$$

The main problem in the above analysis is computing $m_k(w)$. If $\{Z^k(t), t \geq 0\}$ can be modeled as a stationary and ergodic process with state space $\mathcal{S}$ and stationary probability vector, π, then

$$m_k(w) = \log\{\sum_{j \epsilon \mathcal{S}} \pi_k^j \, e^{w \, r^k(j)}\}. \tag{20}$$

SMP Bounds

Consider the case when $\{Z^k(t), t \geq 0\}$ $(k = 1, 2, \ldots, K)$ are independent semi-Markov processes (SMPs) with state space $\mathcal{S}_k = \{1, 2, \ldots, \ell_k\}$ and kernel $G^k(x) = [G^k_{ij}(x)]$. The expected time the k^{th} SMP spends in state i is τ_i^k. The stationary distribution of the k^{th} SMP $\{Z^k(t), t \geq 0\}$ is p^k, where

$$p_i^k = \lim_{t \to \infty} P\{Z^k(t) = i\}.$$

First the computation of $eb_k(v)$ is described. Let $\tilde{G}_{ij}(s)$ be the Laplace Stieltjes transform (LST) of $G_{ij}(x)$. For a given $v > 0$, define

$$\chi^k_{ij}(v, u) = \tilde{G}^k_{ij}(-v(r_k(i) - u)),$$

$$\chi^k(v, u) = [\chi^k_{ij}(v, u)].$$

Then $eb_k(v)$ is given by the smallest positive number such that the Perron Frobenius eigenvalue of $\chi^k(v, eb_k(v))$ is one. Let η be a solution to Equation (15), and denote $\Phi(\eta) = \chi(\eta, eb_k(\eta))$. Let h^k be the left eigenvector of $\Phi(\eta)$ corresponding to the eigenvalue 1, i.e.,

$$h^k = h^k \Phi^k(\eta).$$

Also,

$$P^k(i, j) = [G^k(\infty)]_{ij}. \tag{21}$$

Also define

$$H^k = \sum_{i=1}^{\ell_k} \frac{h_i^k}{\eta(r_k(i) - eb_k(\eta))} \left(\sum_{j=1}^{\ell_k} (\phi^k_{ij}(\eta)) - 1 \right), \tag{22}$$

$$\Psi^k_{min}(i, j) = \inf_x \left\{ \frac{h_i^k e^{-\eta(r_k(i) - eb_k(\eta))x} \int_x^\infty e^{\eta(r_k(i) - eb_k(\eta))y} dG^k_{ij}(y)}{\frac{p_i^k}{\tau_i^k} \int_x^\infty dG^k_{ij}(y)} \right\}, \tag{23}$$

and

$$\Psi^k_{max}(i, j) = \sup_x \left\{ \frac{h_i^k e^{-\eta(r_k(i) - eb_k(\eta))x} \int_x^\infty e^{\eta(r_k(i) - eb_k(\eta))y} dG^k_{ij}(y)}{\frac{p_i^k}{\tau_i^k} \int_x^\infty dG^k_{ij}(y)} \right\}. \tag{24}$$

From Gautam et al [31],

$$C_* e^{-\eta x} \leq P(X > x) \leq C^* e^{-\eta x}, \quad x \geq 0, \tag{25}$$

where

$$C^* = \frac{\prod_{k=1}^{K} H^k}{\min_{\mathcal{A}} \prod_{k=1}^{K} \Psi_{min}^k(i_k, j_k)}, \quad C_* = \frac{\prod_{k=1}^{K} H^k}{\max_{\mathcal{A}} \prod_{k=1}^{K} \Psi_{max}^k(i_k, j_k)},$$

$$\mathcal{A} = \left\{ (i_1, j_1), (i_2, j_2), \ldots, (i_K, j_K) : i_k, j_k \in \mathcal{S}_k, \sum_{k=1}^{K} r_k(i_k) > c \text{ and } \forall k,\ P^k(i_k, j_k) > 0 \right\}. \tag{26}$$

Computation of Ψ_{max} and Ψ_{min} :

Consider a nonnegative random variable Y with distribution $G_{ij}(x)/G_{ij}(\infty)$ and density

$$g_{ij}(x) = \frac{dG_{ij}(x)}{dx} \frac{1}{G_{ij}(\infty)}.$$

The failure rate function of Y is defined by

$$\lambda_{ij}(x) = \frac{g_{ij}(x)}{1 - \frac{G_{ij}(x)}{G_{ij}(\infty)}}. \tag{27}$$

Y is said to be an increasing failure rate (IFR) random variable if

$$\lambda_{ij}(x) \uparrow x$$

and Y is said to be a decreasing failure rate (DFR) random variable if

$$\lambda_{ij}(x) \downarrow x.$$

It is possible to obtain closed form algebraic expressions for $\Psi_{max}(i,j)$ and $\Psi_{min}(i,j)$ in Equations (24) and (23) respectively if a random variable Y with distribution $G_{ij}(x)/G_{ij}(\infty)$ is an IFR or DFR random variable. The following notation is used to compute $\Psi_{max}(i,j)$ and $\Psi_{min}(i,j)$ in those cases. Let x^* and x_* be such that

$$x^* = \arg\sup_x \left\{ \frac{h_i \int_x^\infty e^{\eta(r_i - c)y} dG_{ij}(y)}{\frac{p_i}{\tau_i} e^{\eta(r_i - c)x} \int_x^\infty dG_{ij}(y)} \right\} \tag{28}$$

and

$$x_* = \arg\inf_x \left\{ \frac{h_i \int_x^\infty e^{\eta(r_i - c)y} dG_{ij}(y)}{\frac{p_i}{\tau_i} e^{\eta(r_i - c)x} \int_x^\infty dG_{ij}(y)} \right\}. \tag{29}$$

If Y is IFR or DFR, then $\Psi_{max}(i,j)$ and $\Psi_{min}(i,j)$ in Equations (24) and (23) respectively occur at x values given by the following table

	IFR		DFR	
	$r_i > c$	$r_i \leq c$	$r_i > c$	$r_i \leq c$
x^*	0	∞	∞	0
$\Psi_{max}(i,j)$	$\frac{\phi_{ij}(-\eta(r_i-c))\tau_i h_i}{p_{ij}p_i}$	$\frac{\tau_i h_i \lambda_{ij}(\infty)}{p_i(\lambda_{ij}(\infty)-\eta(r_i-c))}$	$\frac{\tau_i h_i \lambda_{ij}(\infty)}{p_i(\lambda_{ij}(\infty)-\eta(r_i-c))}$	$\frac{\phi_{ij}(-\eta(r_i-c))\tau_i h_i}{p_{ij}p_i}$
x_*	∞	0	0	∞
$\Psi_{min}(i,j)$	$\frac{\tau_i h_i \lambda_{ij}(\infty)}{p_i(\lambda_{ij}(\infty)-\eta(r_i-c))}$	$\frac{\phi_{ij}(-\eta(r_i-c))\tau_i h_i}{p_{ij}p_i}$	$\frac{\phi_{ij}(-\eta(r_i-c))\tau_i h_i}{p_{ij}p_i}$	$\frac{\tau_i h_i \lambda_{ij}(\infty)}{p_i(\lambda_{ij}(\infty)-\eta(r_i-c))}$

where

$$\lambda_{ij}(\infty) = \lim_{x\to\infty} \lambda_{ij}(x).$$

3.2 Multiple Class (Single) Node Models With Multiplexing

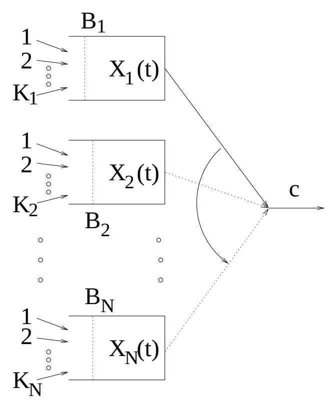

Figure 4: The Multi-class Node Model

In this section the single class model results obtained in Section 3.1 to solve scenarios in multi-class nodes by making suitable transformations. Consider the model of a multi-class node illustrated in Figure 4. The node consists of N input buffers, one for each class of traffic. The input to buffer j ($j = 1, \ldots, N$), is from the K_j sources of class j. The ith source of class j is driven by an independent random environment process $Z_{ij} = \{Z_{ij}(t), t \geq 0\}$ for $i = 1, 2, \ldots, K_j$. At time t, source i of type j generates fluid at rate $r_{ij}(Z_{ij}(t))$. Let $X_j(t)$ be the amount of fluid in buffer j at time t. All the classes of fluids are served by a single channel of constant capacity c, using a specified service scheduling policy. Three policies are studied here: timed round robin (polling) policy, static priority service policy, and, generalized processor sharing (GPS) policy.

Assume that all N buffers are of infinite capacity. If B_j is the actual size of buffer j ($j = 1, 2, \ldots, N$), then

$$\lim_{t\to\infty} P\{X_j(t) > B_j\} = P\{X_j > B_j\}$$

is the steady state approximation of the overflow probability from buffer j. Let ϵ_j be the cell loss probability target for class j traffic ($j = 1, 2, \ldots, N$). The Quality of Service (QoS) criterion for loss probability that need to be satisfied class j traffic is

$$\lim_{t\to\infty} P\{X_j(t) > B_j\} = P\{X_j > B_j\} < \epsilon_j. \tag{30}$$

Note that although bounds can be obtained for the delay, explicit expressions for delay QoS is a research issue that needs to be addressed. Also, delay-jitter QoS measures are research problems to be studied.

The three service scheduling policies, timed round robin policy, static priority service policy, and, GPS policy are discussed. Note that the effective bandwidth and the SMP bounds analysis for the multiclass model is not a trivial extension of that of the single class model. The output channel capacity for each buffer is not a constant in the multiclass node model. Therefore the model requires a careful transformation that results in a constant output channel capacity model for each of the buffers. From the transformed models, $P\{X_j > B_j\}$ needs to be computed.

3.2.1 Timed Round Robin (Polling)

Consider the multi-class node model described in Section 3.2 and illustrated in Figure 4. All classes of fluids are multiplexed using a *Timed Round Robin* service scheduling policy which is described as follows. The scheduler allocates the entire output capacity c to each of the N buffers in a cyclic fashion. In each cycle, buffer j gets the entire capacity for an interval of length τ_j. Note that during this interval, buffer j could be empty. Hence the scheduler is not work conserving.

Let t_{so} be the total switch-over time during an entire cycle. Assume that t_{so} does not change with time. The *cycle time* T is defined as the amount of time the scheduler takes to complete a cycle, and is given by

$$T = t_{so} + \sum_{j=1}^{N} \tau_j. \tag{31}$$

First assume that all buffers are of infinite capacity. The dynamics of the buffer-content process $\{X_j(t), t \geq 0\}$ is described by

$$\frac{dX_j(t)}{dt} = \begin{cases} \sum_{i=1}^{K_j} r_{ij}(Z_{ij}(t)) - c & \text{if } X(t) > 0 \text{ and scheduler serving buffer } j \\ \left\{\sum_{i=1}^{K_j} r_{ij}(Z_{ij}(t)) - c\right\}^+ & \text{if } X(t) = 0 \text{ and scheduler serving buffer } j \\ \sum_{i=1}^{K_j} r_{ij}(Z_{ij}(t)) & \text{if scheduler not serving buffer } j. \end{cases} \tag{32}$$

Assume that the following stability condition is satisfied for buffer j, $(j = 1, \ldots, N)$

$$\sum_{i=1}^{K_j} E\{r_{ij}(Z_{ij}(\infty))\} < c\frac{\tau_j}{T}. \tag{33}$$

Effective Bandwidth Analysis

If given τ_1, τ_2, ..., τ_N and t_{so} are given, then the buffer contents of a given buffer (say j) and its dynamics do not depend on the parameters of any other buffer (say $i \neq j$). Therefore, it is convenient to analyze each buffer separately. Buffer j can be modeled as a single-buffer-fluid model with variable output capacity and input from K_j different sources, such that source i of class j is modulated by an environmental process $\{Z_{ij}(t), t \geq 0\}$. The output capacity alternates between c (for τ_j units of time) and 0 (for $T - \tau_j$ units of time).

Note that the effective-bandwidth approximation and the SMP bounds assume that the output channel capacity is a constant. Therefore to utilize those techniques, one needs to first transform the model into an appropriate one with a constant output channel capacity as follows :

Consider a single-buffer-fluid model for buffer j with a constant output channel capacity c whose input is generated by the original K_j sources and a fictitious compensating source. The compensating source is such that it stays on for a deterministic amount of time $T - \tau_j$ and off for a deterministic amount of time τ_j. When the compensating source is on, it generates fluid at rate c and when it is off it generates fluid at rate 0. Note that the compensating source is independent of the original K_j sources. Clearly, the dynamics of the buffer-content process (of buffer j) in Equation (32) remain unchanged for this transformed single-buffer-fluid model with $K_j + 1$ input sources (including the compensating source) and constant output capacity c. Refer to Figure 5 for an illustration of the transformed model for buffer j.

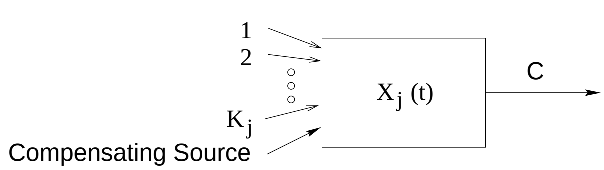

Figure 5: Transformed Buffer j Model

Using the effective bandwidth computations in Kulkarni [50], one can show that the effective bandwidth of the compensating source described above is given by

$$eb_j^s(v) = \frac{c(T - \tau_j)}{T}. \tag{34}$$

Note that the effective bandwidth of this deterministic source is indeed its mean traffic generation rate. Let the effective bandwidth of source i ($i = 1, 2, \ldots, K_j$) of class j be $eb_{ij}(v)$. Therefore $P(X_j > B_j) \approx e^{-B_j \eta_j}$, where η_j (using Equation (15)) is obtained by solving

$$\sum_{i=1}^{K_j} eb_{ij}(\eta_j) + c\frac{(T - \tau_j)}{T} = c. \tag{35}$$

The loss probability QoS criteria for all the classes of traffic are satisfied if for all $j = 1, 2, \ldots, N$,

$$e^{-B_j \eta_j} < \epsilon_j. \tag{36}$$

Hence Equation (35) indicates that the QoS guarantee using the effective-bandwidth approximation technique depends only on the ratio τ_j / T and not the individual values of τ_j or T. Consider two instances, one with large τ_j and T and the other with small τ_j and T, such that the ratio τ_j / T is the same in both instances. The effective bandwidth approximation implies that the loss probability will be less than ϵ_j in both instances. This goes against intuition. It is theoretically valid since the effective-bandwidth analysis assumes that $B_j \to \infty$. However in practice, this cannot be valid due to finite buffers.

Therefore the effective-bandwidth approximation technique fails for moderate to large sized buffers and works only for extremely large sized buffers! The Chernoff dominant eigenvalue approximation (see Elwalid and Mitra [28]) also faces the same problem. The SMP bounds below resolve this issue.

Semi-Markov Process (SMP) Bounds Analysis

Consider the transformed model of buffer j ($j = 1, 2, \ldots, N$) illustrated in Figure 5. Assume that the $\{Z_{ij}(t), t \geq 0\}$ processes ($i = 1, 2, \ldots, K_j$) are semi-Markov processes. Therefore there are $K_j + 1$ independent sources modulated by SMPs (including the compensating source) that generate traffic into buffer j whose the output capacity is a constant c.

For the SMP bounds analysis for buffer j, follow the single-class traffic analysis in Section 3.1.2 for a buffer with input generated by independent semi-Markovian sources multiplexed together. Let η_j be the smallest positive solution to Equation (35).

Using Equations (22), (23) and (24), one can obtain H^{ij}, Ψ^{ij}_{min} and Ψ^{ij}_{max} respectively for source i ($i = 1, 2, \ldots, K_j$) of class j. The corresponding expressions H^{sj}, Ψ^{sj}_{min} and Ψ^{sj}_{max} for the jth compensating source are

$$H^{sj} = \frac{1-\exp\left(-\eta_j c \frac{T-\tau_j}{T}\tau_j\right)}{\eta_j c}\left[\frac{T^2}{(T-\tau_j)\tau_j}\right], \tag{37}$$

$$\Psi^{sj}_{min} = \begin{bmatrix} 0 & T\exp\left(-\eta_j c \frac{T-\tau_j}{T}\tau_j\right) \\ T\exp\left(-\eta_j c \frac{T-\tau_j}{T}\tau_j\right) & 0 \end{bmatrix}, \tag{38}$$

$$\Psi^{sj}_{max} = \begin{bmatrix} 0 & T \\ T & 0 \end{bmatrix}. \tag{39}$$

Letting $s = K_j + 1$, the bounds on the limiting distribution of the buffer content process $\{X_j(t), t \geq 0\}$ as

$$C_{j*}e^{-\eta_j x} \leq P(X_j > x) \leq C_j^* e^{-\eta_j x},$$

where, η_j is from Equation (35),

$$C_j^* = \frac{\prod_{k=1}^{K_j+1} H^{kj}}{\min_{\mathcal{A}^j}\prod_{k=1}^{K_j+1}\Psi^{kj}_{min}(l_k, m_k)}, \tag{40}$$

$$C_{*j} = \frac{\prod_{k=1}^{K_j+1} H^{kj}}{\max_{\mathcal{A}^j}\prod_{k=1}^{K_j+1}\Psi^{kj}_{max}(l_k, m_k)}, \tag{41}$$

and

$$\mathcal{A}^j = \left\{(l_1, m_1), (l_2, m_2), \ldots, (l_{K_j+1}, m_{K_j+1}) : l_k, m_k \in \mathcal{S}_k, \sum_{k=1}^{K_j+1} r_{kj}(l_k) > c \text{ and } \forall k,\ P^{kj}(l_k, m_k) > 0\right\}. \tag{42}$$

The QoS criteria for all the classes of traffic are satisfied if, for $j = 1, 2, \ldots, N$,

$$C_j^* e^{-\eta_j B_j} < \epsilon_j. \tag{43}$$

Clearly, H^{sj} and Ψ^{sj}_{min} are functions of τ_j, T and τ_j/T. Hence, C_j^* is a function of both τ_j and T and not simply of the ratio τ_j/T.

3.2.2 Static Priority Service Policy

In this section, consider a *Static Priority Service Policy* (for the model in Section 3.2 and illustrated in Figure 4) to multiplex the multi-class traffic which operates as follows. Under this service policy, traffic of class j has higher service priority over traffic of class i, if $i > j$. The scheduler serves the traffic of class j only if there is no fluid of higher priority in the buffers. Thus all the available channel capacity (a maximum of c) is assigned for the class-1 fluid and the leftover channel capacity (if any) that class-1 does not need, to class-2 fluid.

Any leftover channel capacity that class-1 and class-2 do not need, is assigned to class-3 fluid, and so on.

For a comprehensive study on effective bandwidths with priorities, see Berger and Whitt [8] and [9]. See Gautam and Kulkarni [52] for the study of effective bandwidth and CDE approximations for the static priority case. Here a two-class traffic case is explained, although the analysis can be extended to more than 2 classes. The K_j class-j sources, $j = 1, 2$, are independent and identical on-off sources with exponential on and off times, on-time parameter α_j, off-time parameter β_j and on-time rate r_j.

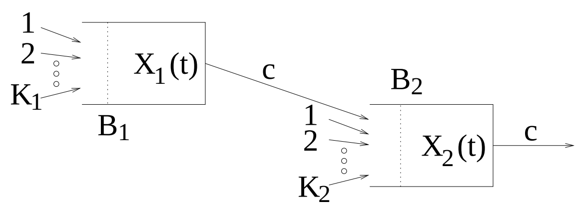

Figure 6: The Transformed Model

Consider the transformed model in Figure 6. The sample paths of the buffer content processes $\{X_1(t), t \geq 0\}$ and $\{X_2(t), t \geq 0\}$ in this model are identical to those in the original model in Figure 4 (for $N = 2$). This observation is made in Elwalid and Mitra [28] and is immensely useful in the analysis. Note that the output from buffer 1 can be modeled as an SMP. Hence, the input to buffers 1 and 2 can be modeled as ones with multiplexing independent SMP sources.

Buffer 1 : If $K_1 \leq c/r_1$, then $P\{X_1 > B_1\} = 0$, since buffer 1 will always be empty. Now for the case $K_1 > c/r_1$, the steady-state distribution of the buffer-content process is bounded as

$$C_{*1}e^{-\eta_1 B_1} \leq P\{X_1 > B_1\} \leq C_1^* e^{-\eta_1 B_1},$$

where

$$\eta_1 = \frac{K_1(c\alpha_1 + c\beta_1 - K_1\beta_1 r_1)}{c(K_1 r_1 - c)}. \tag{44}$$

$$C_1^* = \frac{\left(\frac{K_1 r_1}{K_1 r_1 - c}\frac{\alpha_1}{\alpha_1+\beta_1}\right)^{K_1}}{\left(\frac{c\alpha_1}{\beta_1(K_1 r_1 - c)}\right)^{\lceil \frac{c}{r_1} \rceil}},$$

and

$$C_{*1} = \left(\frac{K_1 r_1 \beta_1}{c(\alpha_1 + \beta_1)}\right)^{K_1}.$$

Buffer 2 : First model the K_2 exponential on-off sources as a single (K_2+1)-state SMP with the states denoting the number of priority-2 sources that are on and then derive expressions for H^1, $\Psi^1_{max}(i,j)$ and $\Psi^1_{min}(i,j)$ as defined in Equations (22), (23) and (24). In Kulkarni and Gautam [52] it is shown that the output process from buffer 1 can be modeled as an SMP. The corresponding expressions H^2, $\Psi^2_{max}(i,j)$ and $\Psi^2_{min}(i,j)$ for the SMP model of the output from buffer 1 can be derived. Therefore one can analyze the input to buffer 2 as traffic from two sources (output from buffer 1 and the (K_2+1)-state SMP), each modulated by an SMP.

Begin by obtaining η_2. Note that η_2 solves either

$$K_1\, eb_1(\eta_2) + K_2\, eb_2(\eta_2) = c \quad \text{and } \eta_2 \le v^* \tag{45}$$

or

$$\frac{v^*}{\eta_2} K_1\, eb_1(v^*) + K_2\, eb_2(\eta_2) = \frac{cv^*}{\eta_2} \;\; \text{and } \eta_2 > v^*, \tag{46}$$

where

$$v^* = \frac{\beta_1}{r_1}\left(\sqrt{\frac{c\alpha_1}{\beta_1(K_1 r_1 - c)}} - 1\right) + \frac{\alpha_1}{r_1}\left(1 - \sqrt{\frac{\beta_1(K_1 r_1 - c)}{c\alpha_1}}\right),$$

and for $j = 1, 2$

$$eb_j(v) = \frac{r_j v - \alpha_j - \beta_j + \sqrt{(r_j v - \alpha_j - \beta_j)^2 + 4\beta_j r_j v}}{2v}. \tag{47}$$

Therefore using the expressions for H^1, $\Psi^1_{max}(i,j)$, $\Psi^1_{min}(i,j)$, H^2, $\Psi^2_{max}(i,j)$ and $\Psi^2_{min}(i,j)$ we have

$$C_2^* = \frac{H^1 H^2}{\min_{(i_1,j_1),(i_2,j_2):\min\{i_1 r_1, c\} + i_2 r_2 > c, p_{i_1 j_1} > 0, p_{i_2 j_2} > 0} \Psi^1_{min}(i_1, j_1)\ \Psi^2_{min}(i_2, j_2)}$$

and

$$C_{*2} = \frac{H^1 H^2}{\max_{(i_1,j_1),(i_2,j_2):\min\{i_1 r_1, c\} + i_2 r_2 > c, p_{i_1 j_1} > 0, p_{i_2 j_2} > 0} \Psi^1_{max}(i_1, j_1)\ \Psi^2_{max}(i_2, j_2)}.$$

3.2.3 Generalized Processor Sharing (GPS)

Consider the multi-class node model described in Section 3.2 and illustrated in Figure 4. All classes of fluids are multiplexed using a *generalized processor sharing* service scheduling policy which is described in the following manner. Consider preassigned numbers ϕ_1, ϕ_2, ..., ϕ_N for each of the N classes numbers such that if all the input buffers have traffic entering, the scheduler allocates output capacity c in the ratio $\phi_1 : \phi_2 : \ldots : \phi_N$ to each of the N

buffers. If one or more of the buffers are empty and no traffic enters those buffers, then the capacity c is divided in the ratio of ϕ_j's of the remaining buffers.

The discrete version of the GPS is called the Packetized General Processor Sharing (PGPS). The PGPS service policy is based on the generalized processor sharing approach explained in Parekh and Gallager [66]. The Quality-of-Service aspects, effective bandwidths, admission control, etc for the GPS and PGPS have been addressed in detail in de Veciana et al [24] and [25]. The PGPS is also known as "weighted fair queueing" in the literature.

3.3 Network of nodes models with single and multiple class models

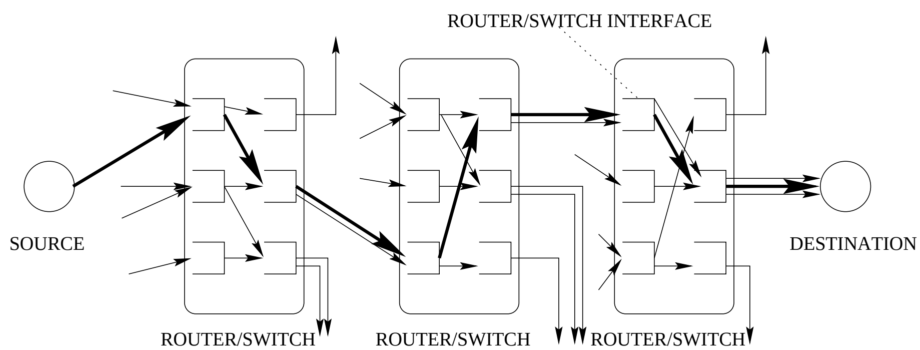

Figure 7: Example Scenario

The scenario for this section is depicted in Figure 7. Consider data flowing from a source to a destination through routers and switches in a high speed network handling multiple classes of traffic. The different classes of traffic are differentiated according to the priorities they receive. For example critical information could receive very high priority, latency sensitive applications moderately high priority, and latency insensitive applications such as email very low priority. When packets belonging to different classes or priorities arrive at a router or switch interface, the packets are flushed out of the interface buffer using a given scheduling mechanism (such as static priority, round-robin polling, generalized processor sharing, etc). Traffic belonging to all classes are multiplexed when they leave a router or switch interface. However when the traffic stream encounters another router or switch interface downstream, it gets demultiplexed into its original classes. The objective of the network is to provide guaranteed end-to-end QoS.

For the single class case, the following is the analysis for determining the effective bandwidth of the output traffic from a buffer. The output traffic from a buffer acts as input traffic for a downstream node in a network. Typically, it may not be possible to characterize some output processes as tractable stochastic processes and compute the effective bandwidths. Consider a network with nodes in tandem where the output from one buffer acts as input to another buffer. Clearly, the output cannot be characterized by simple processes like a CTMC. Although it can be shown to be a regenerative source, the characterization is intractable.

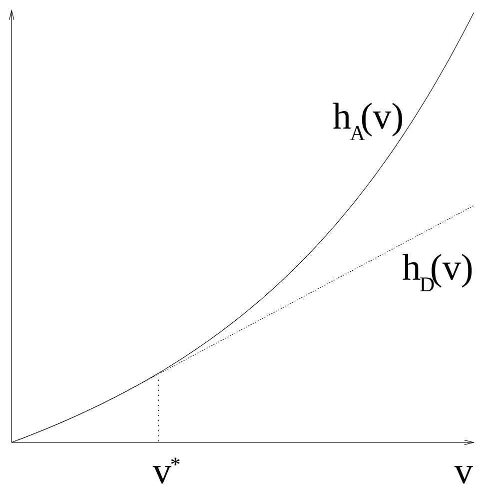

Figure 8: $h_A(v)$ and $h_D(v)$ vs v

Chang and Thomas [15], Chang and Zajic [16] and de Veciana et al [22], derive the effective bandwidths of the output of a node in terms of the input source. Let $A(t)$ be the total input to the buffer over $(0, t]$ and $D(t)$ be the total output from the buffer of a single-buffer-fluid model over $(0, t]$. The capacity of the output channel is c. Analogous to Equation (5), define the ALMGF of the output as

$$h_D(v) = \lim_{t \to \infty} \frac{1}{t} \log E\{\exp(vD(t))\}. \tag{48}$$

If $h_A(v)$ is the ALMGF of the input to a buffer then the ALMGF of the output (of capacity c) from the buffer is given in terms of $h_A(v)$ by

$$h_D(v) = \begin{cases} h_A(v) & \text{if } 0 \leq v \leq v^* \\ h_A(v^*) - cv^* + cv & \text{if } v > v^*, \end{cases} \tag{49}$$

where v^* is obtained by solving for v in the equation,

$$\frac{d}{dv}\left[h_A(v)\right] = c.$$

Figure 8 illustrates the relationship between $h_A(v)$ and $h_D(v)$. The QoS problem for different types of single-class networks are studied by de Veciana et al [22] and [24] and

Gautam [33]. Essentially the sum of the effective bandwidths of all the traffic sources into a buffer should be compared to the output capacity. Also since the route taken by a traffic stream is known, it is easy to use the effective bandwidth of the output from a node to derive the effective bandwidth of a downstream node.

One of the important research issues is to obtain end-to-end performance measures in multi-class networks. Since the multiple classes of traffic use a common service scheduling mechanism that would result in a low volume of a particular class and high volume of traffic of another class. Intuitively the multiple classes of traffic are negatively correlated. This could potentially be used to obtain conservative estimates of QoS measures by exploiting stochastic monotonicity properties. Using the results in Puhalskii and Whitt [67] for functional large deviations principle for waiting and departure processes, it is possible to obtain the required performance measures.

4 LAN (multiaccess communication) models

4.1 Slotted and Unslotted Aloha

One of the foremost multiaccess communication protocol is the Aloha, developed in the University of Hawaii. The following stochastic models for slotted and unslotted Aloha are adapted from Kulkarni [51]. Assume that there are N users at geographically diverse locations that transmit messages (in the form of packets) via satellites. In the **slotted Aloha** version, it is assumed that the clocks of all users are synchronized. Therefore at time slots $n = 1, 2, 3, \ldots$, each user, independent of other users, transmits a packet with probability p. If more than one user transmits a packet during a given slot, then a collision results between all the packets and the resulting message is garbled. All the users involved in a collision retransmit at the beginning of a slot with probability r, however, if a user has a message to retransmit no new messages are transmitted by this user. If a user has a packet to retransmitted, then this users is termed a "backlogged" user. Let X_n be the number of backlogged users at the beginning of the n^{th} slot. Clearly there will be $N - X_n$ unbacklogged users at the beginning of the n^{th} slot. It can be shown that the process $\{X_n, n \geq 0\}$ is a DTMC (as shown in Kulkarni [51]) with transition probability matrix that can be derived using:

$$P\{X_{n+1} = i - 1 | X_n = i\} \;\; = \;\; (1-p)^{N-i} i \, r \, (1-r)^{i-1}$$

$$
\begin{aligned}
P\{X_{n+1} = i+1 | X_n = i\} &= (N-i)p(1-p)^{N-i-1}(1-(1-r)^i) \\
P\{X_{n+1} = i+j | X_n = i\} &= \binom{N-i}{j} p^j (1-p)^{N-i-j} \quad 2 \le j \le N-i \\
P\{X_{n+1} = i | X_n = i\} &= (N-i)p(1-p)^{N-i-1}(1-r)^i + (1-p)^{N-i}(1-i\,r\,(1-r)^{i-1}).
\end{aligned}
$$

There are several modifications to the slotted Aloha that have eventually resulted in efficient satellite communications.

One of the **unslotted Aloha** versions considers a system where each user, when not backlogged, generates messages according to a Poisson process. Message transmission times are according to an exponential distribution. A collision results when a user attempts to transmit while another user is transmitting. If a collision results, all transmissions are terminated instantaneously. All messages involved in a collision wait for an exponential time before attempting to retransmit. If $X(t)$ denotes the number of backlogged messages at time t and $Y(t)$ a binary variable that denotes whether or not a message is under transmission at time t. It is possible to model $\{(X(t), Y(t)), t \ge 0\}$ as a CTMC. Using the steady state probability distributions of the DTMC for the slotted Aloha and the CTMC for the unslotted Aloha, performance measures such as throughput (expected number of successful transmissions per unit time), satellite utilization, expected number of backlogged messages, expected delay in successfully transmitting a message, etc can be computed. Using the performance measures it is possible to derive optimal designs for the Aloha systems.

4.2 Ethernet models

The most popular local area network (LAN) is the Ethernet (see Walrand and Varaiya [77]). The popularity is due to the high performance and low cost. The protocol used in Ethernet is CSMA/CD (Carrier Sense Multiple Access with Collision Detection). The following is a simplified model of an Ethernet where there are a number of identical nodes (say N) connected onto a common cable. A significant portion of the following description is adapted from Bertsekas and Gallager [10].

When one node transmits a packet (and the others are silent), all the other nodes hear that packet. In addition, a node can listen to the cable before transmitting (i.e., conceptually, 0, 1, and idle can be distinguished). Finally, because of the physical properties of the cable, it is possible for a node to listen to the cable while transmitting. Thus, if two nodes start to transmit almost simultaneously, they will shortly detect a collision in process and both cease transmitting. This technique is called CSMA/CD. On the other hand, if one node

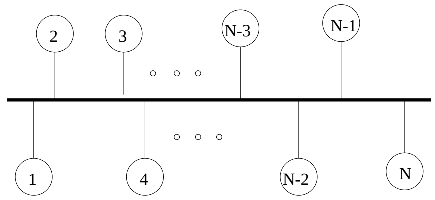

Figure 9: A simplified model for Ethernets

starts transmitting and no other node starts before the first node's signal has propagated throughout the cable, the first node is guaranteed to finish its packet without collision. Thus, the first portion of a packet can be viewed as making a reservation for the rest of the packet. **Slotted CSMA/CD in Ethernet, DTMC model**: For analytic purposes, it is easier to visualize Ethernet in terms of megaslots and minislots. The minislots are of duration β, which denotes the time required for a signal to propagate from one end of the cable to the other and to be detected. If the nodes are all synchronized into minislots of this duration, and if only one node transmits in a minislot, all other nodes will detect the transmission and not use subsequent minislots until the entire packet is completed. If more than one node transmits in a minislot, each transmitting node will detect the condition by the end of the minislot and cease transmitting. Packets or messages are backlogged for retransmission when there is a collision.

To model the system, first assume that the minislots could be of 3 types – no transmission (idle), one transmission (one) and many transmissions (many). An *idle-minislot* is followed by another *idle-minislot* if none of the users (backlogged or non-backlogged) decide to transmit. An *idle-minislot* is followed by a *many-minislot* if many of the users (backlogged or non-backlogged) decide to transmit which results in a collision and messages are backlogged. An *idle-minislot* is followed by a *one-minislot* if exactly one of the users (backlogged or non-backlogged) decides to transmit. A *one-minislot* is always followed by a megaslot (when the entire packet is transmitted without collision). A *many-minislot* is always followed by an *idle-minislot*. All minislots (idle, one and many) are of duration β. The megaslot is of random duration with mean α (here it is assumed that megaslots are of constant duration α) during which only one message is transmitted successfully. A megaslot is always followed by an *idle-minislot*. It is assumed that at the end of an *idle-minislot*, each backlogged user will attempt to retransmit with probability r and each non-backlogged user will attempt to transmit with probability p. The system can be modeled as a DTMC where the state of the DTMC at the end of a slot is the number of backlogged users and the type of slot (idle, one, many, mega).

Unslotted CSMA/CD in Ethernet, CTMC model: Messages (packets) are generated by each of the N nodes according to Poisson processes. As soon as a message (packet) is generated, the node attempts to transmit it onto the cable. If the node detects the transmission of another packet during the attempt, it withdraws the attempt and this packet is backlogged. There are also backlogged packets whenever a collision occurs. If the node does not detect the transmission of another packet and there are no other packets starting to transmit then this packet begins transmission starting with an initial phase. If during the initial phase (analogous to a minislot) there are no collisions then the packet is transmitted successfully in the final phase. Assume that during the initial phase of transmission, none of the other nodes can detect the packet being transmitted whereas during the final phase all nodes can detect packet transmission. Also assume that collisions that occur during the initial phase can be immediately detected! Note that the initial and final phases are each exponentially distributed. All backlogged packets wait for a random amount of time (distributed exponentially with mean $1/\theta$) before retransmission. The system can be modeled as a CTMC where the state of the CTMC at the end of a slot is the number of backlogged users and the type of slot (idle, initial and final).

For both the DTMC and the CTMC models, performance measures such as throughput (expected number of successful transmissions per unit time), cable utilization, expected number of backlogged messages, expected delay in successfully transmitting a message, etc can be computed. Using the performance measures it is possible to evaluate optimal designs for the Ethernet systems. See Bertsekas and Gallager [10] for modified versions and approximations.

4.3 Token Rings

Besides Ethernet, the other commonly used LAN architecture and protocol is the Token Ring, developed by IBM. Here a simplistic model for a token ring is explained and is based on the description in Roy [72]. Consider N independent and identical users that are arranged logically in the form of a ring (see Figure 10).

Unlike the Ethernet model where all the users are allowed to transmit simultaneously which could potentially result in collisions, the token ring scheme is such that at a time there is at most one user generating a message over the cable or ring (thus there are no collisions).

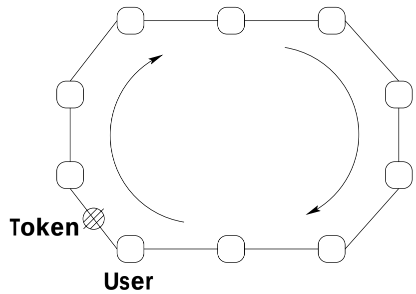

Figure 10: Model of a token ring LAN

A designated user generates a “free” token into the ring. This token traverses the ring in a given direction. When a user with a message to transmit receives the free token, the user holds on to the token (now called “busy” token) and transmits the message onto the ring or cable. There are two basic types of implementation, the exhaustive and the gated service. In the exhaustive service a user that receives a free token transmits packets until there are no packets to transmit, however in the gated service case, only the packets that arrived prior to receiving the free token are transmitted and the packets arriving during the transmission will be transmitted during the next free token arrival to the user! Once the user completes transmission, the busy token is converted into a free token and passed along the ring.

To model the system as a CTMC, one could assume that the packets are generated according to a Poisson process, the length of the packets are exponentially distributed, and, the propagation time (including latency at the user) is also exponentially distributed. Since all the users are identical, a CTMC of the form $\{(X(t), Y(t), t \geq 0\}$ modeled where $X(t)$ is the number of messages in the network and $Y(t)$ is the status of the token (free or busy) at time t. Using the steady state distribution of the CTMC, performance measures such as throughput, delay, and blocking probability (if the users have finite buffers) can be computed.

Realistically speaking users may belong to multiple classes and are not necessarily identical. Also, the exponential distribution may not be the most appropriate. Several researchers have addressed these shortcomings (see Chae and Nilsson [18] for the performance analysis of a prioritized token ring with reservation model) and there are other interesting problems to be addressed in the future.

5 Other Topics And Models

5.1 TCP and flow control

When a message needs to be sent from a source to a destination, it is broken down into small packets and transported from the source to the destination. The protocols responsible for this transport of packets over networks are user datagram protocol (UDP) and transmission control protocol (TCP). Certain applications (typically real-time) use UDP where the destination does not acknowledge the receipt of packets to the source. Therefore in UDP the source does not know if the message sent, firstly reached the destination, and if it did, whether it reached without any losses.

On the other hand, TCP is an acknowledgement (ACK) based protocol. Every packet that reaches the destination is acknowledged. Therefore TCP is useful for applications that cannot tolerate losses, at the same time can tolerate slow transmission. There has been tremendous amount of research in the area of speeding up TCP, and also modeling it for different networks such as ATM, wireless, etc. A simplified version of TCP is explained below. Readers are encouraged to refer to Van Jacobson [44] and [45], Romanow and Floyd [70], Stevens [74], etc for a detailed description.

Instead of waiting for an ACK for every packet sent, the source sends n packets to the destination before receiving an ACK. These n packets constitute the window with n as the window size. The window size is not a constant throughout the connection. If the connection is across a network with low congestion then the window size gradually increases upto a prenegotiated maximum W_{max}. However if congestion develops and packets are lost, TCP backs off the packet generation by reducing the window size to half. The backing off is performed using a timer so that if the source does not receive an ACK before a certain time (this is also variable, in fact it depends on the connection and round trip time between source and destination for packets), the source retransmits with a smaller window.

Marsan et al [60] develop an approximate CTMC model for performance analysis of TCP connections in high-speed ATM networks. The modeling takes into account the slow start (initially the window size increases slowly), fast recovery, and congestion-avoidance (window size reduction) strategies commonly used in TCP. The sparse but regular structure of the infinitesimal generator matrix is taken advantage of in the analysis.

Misra and Ott [62] analyze the stationary behavior of the TCP congestion window. Most of the earlier analysis assumed that the loss probability is constant with respect to window size. With the development of Random Early Detection (RED) it is important to consider the loss probability that varies with respect to the window size (as the window size increases, the loss probability increases in a stochastic sense). A Markov process that is further approximated as a continuous time, continuous state space system is modeled. The stationary distribution of the process is analyzed.

Kumar [55] studies the performance of various TCP versions such as TCP-OldTahoe (uses timeout recovery), TCP-Tahoe (uses fast retransmit), TCP-Reno (uses fast retransmit and fast but conservative recovery), and, TCP-NewReno (uses fast retransmit and fast recovery). A stochastic model (Markov renewal reward process) is used to study the throughput performance of the different TCP versions in the presence of random loss on a wireless link. The main results include the following: TCP-Reno performs no better or worse that TCP-Tahoe for large packet loss probability. TCP-NewReno is a considerable improvement over TCP-Tahoe.

Baccelli and Bonald [7] consider window flow control in lossless packet-switched networks (essentially applicable to TCP). However the window is assumed to be static and all packets follow the same route between a source and destination. There is also exogenous traffic along the route. General stochastic processes that are stationary and ergodic are used to model input processes. The stability of the system is evaluated and performance measures such as bounds on the maximal throughput are obtained.

5.2 Routing

Routing in the current version of the Internet uses best effort schemes and does not use any congestion avoidance mechanisms. The routers in the Internet use a learning process to develop a routing table. Based on the routing table an incoming packet is delivered to the appropriate neighboring router. This procedure continues from the source to the destination. The final source-destination route is usually the minimal hop path. In the case of a breakdown, the routers reconfigure their routing tables appropriately. The benefits to the best effort scheme are: easy implementation, fast learning (or recovery after breakdown), and fairness.

However in circuit-switched networks where the network topology is known and the number of circuits n_i of each link i between a pair of switches is given. Assume that the rate of call requests per unit time between every pair of source and destination is given. Consider a source S and a destination D between which there are R possible routes. Any message between S and D is sent across route j with probability $p_j(SD)$. Therefore when a new connection needs to be established between S and D, a random number is generated and the resulting route is selected. However if that route is not free, the call for connection is blocked or rejected. This is the static routing policy which could potentially result in a large number of rejected calls. Optimal values of $p_j(SD)$ (for $j = 1, \ldots, R$) are selected based on minimizing the total cost.

Gibbens et all [35] developed the dynamic alternate routing (DAR) strategy where the (stochastic) k-shortest paths are obtained between all sources and destinations at all points of time. Clearly the paths vary dynamically. Then calls between a source and destination are routed through the current shortest path or current second shortest path. If both are full then the calls are rejected. In Gibbens et al [36] some of the consequences for dynamic routing schemes for dual- and multi-parented networks (where a call can enter or leave the network in two or more points) are considered. Bounds are obtained for optimal dynamic routing strategies. The robustness is also illustrated. Gibbens and Kelly [37] use stochastic analysis of dynamic routing for classical mathematical programming (optimization) to design networks. The methods used are network flow optimization and Markov decision processes for bounds on dynamic routing strategies. In Dasylva and Srikant [21], non-trivial lower bounds on the lost revenue under any routing scheme in a multi-class loss network are obtained. The bound is used to obtain linear programs which give bounds for sparsely connected networks with multiple classes and alternate routing.

One of the key factors that will enable networks to provide guaranteed QoS is the concept of QoS routing. Suppose an application desires the following QoS measures between the source and destination: maximum loss rate ϵ, maximum delay δ and maximum jitter ρ. Then the QoS routing problem is to send the message from the source to destination by the least expensive route such that the loss rate is less than ϵ, delay is less than δ and jitter is less than ρ, each with probability, say 0.9999. This is an important problem to be addressed and is being actively pursued. See Apostolopoulos et al [5] for a description of QoS routing, cost function, comparisons to best-effort routing, etc.

5.3 Leaky bucket policing

5.3.1 Description

The proliferation of the Internet and its excessive congestion has led researchers working on emerging high-speed telecommunication networks to develop tools to police and control the traffic at the user or source end. These policing mechanisms need to not only ensure that the telecommunication network traffic generated by the sources are kept below a negotiated threshold but also ensure that the users receive the Quality of Service (QoS) that they have been promised. One such policing mechanism is the leaky bucket (see Cidon and Gopal [19], Gu et al [39], Gün et al [40], Vamvakos and Anantharam [75], Butto et al [11], Callegati et al [12], Holtsinger and Perros [43], Sohraby and Sidi [73], Wu and Mark [80], and, Yin and Hluckyj [81]).

A leaky bucket is essentially a credit management mechanism that controls the traffic entering the network. A single or a series of leaky buckets can be used to optimally regulate the source traffic in communication networks (see Anantharam and Konstantopoulos [3]). In the recent literature a few researchers have proposed models to optimally select leaky bucket parameters (see Anantharam and Konstantopoulos [1], Raha et al [69], Elwalid and Mitra [29], Gorinsky et al [38], de Veciana [23], etc).

Here stochastic fluid-flow models are used to describe the traffic flow, following the large literature using fluid-flow models for communication systems (see Anick et al [4], Elwalid and Mitra [26], etc). Chen and Yao [13] and [14], Ott and Shanthikumar [65], Harrison [41], Chen and Mandelbaum [17], etc, demonstrate how to convert any discrete arrival system into a fluid-flow system and apply the fluid-flow model results.

"Leaky Bucket" is a control mechanism for admitting data into a network. It consists of a data buffer and a token pool as shown in Figure 11. Use a fluid-flow leaky bucket model assuming that the data traffic and tokens can be modeled as fluids. Tokens are generated continuously at a fixed rate γ into the token pool of size M. The new tokens are discarded if the token pool is full. External data traffic enters the data buffer (of size B_D) from a source modulated by an environmental process $\{Z(t), t \geq 0\}$. Traffic is generated by this source at rate $r(Z(t))$ at time t.

If there are tokens in the token pool, the incoming fluid takes an equal amount of tokens and enters the network. Broadly, there are two types of leaky buckets, the buffered and the unbuffered leaky buckets. If the token pool is empty then two alternative implementations

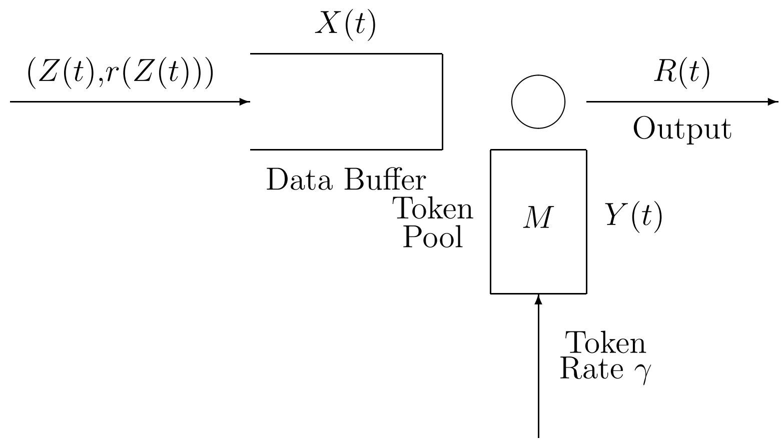

Figure 11: Fluid model of a leaky bucket

are considered:

- Buffered Leaky Bucket: the packets wait in the infinite capacity data buffer ($B_D = \infty$) for tokens to arrive,
- Unbuffered Leaky Bucket: there is no data buffer ($B_D = 0$) for the packets and any packet that does not find a token enters the network carrying a "violation" tag. Later such violation traffic can be dropped if congestion develops.

The buffered and unbuffered leaky bucket models are described in the following sections, and their respective output processes are studied.

5.3.2 Buffered Leaky buckets

The output from a buffered leaky bucket acts as an input to a downstream network node. Hence, in this section the output from the leaky bucket is characterized. Refer to Figure 11. Let $X(t)$ be the amount of traffic in the data buffer at time t. Let $Y(t)$ be the amount of tokens in the token pool at time t ($Y(t) \leq M$). Note that fluid starts accumulating in the data buffer ($X(t) > 0$) only when the token pool is empty ($Y(t) = 0$). As long as tokens are available ($Y(t) > 0$), fluid does not wait at the data buffer ($X(t) = 0$). Therefore $X(t)Y(t) = 0$, for all t. Clearly, when the token pool is not empty ($Y(t) > 0$), the output from the leaky bucket is at rate $r(Z(t))$ at time t and when the token pool is empty, the output from the leaky bucket is at rate γ. Hence the output rate from the leaky bucket at time t, $R(t)$, is given by

$$R(t) = \begin{cases} \gamma & \text{if } Y(t) = 0 \\ r(Z(t)) & \text{if } Y(t) > 0. \end{cases} \quad (50)$$

Define a process $\{W(t), t \geq 0\}$ (see Anantharam and Konstantopoulos [2]) as

$$W(t) = X(t) + M - Y(t). \quad (51)$$

First characterize the $\{W(t), t \geq 0\}$ process. The dynamics of the $X(t)$ and the $Y(t)$ processes are given by

$$\frac{dX(t)}{dt} = \begin{cases} r(Z(t)) - \gamma & \text{if } X(t) > 0 \\ 0 & \text{if } X(t) = 0, \end{cases} \quad (52)$$

$$\frac{dY(t)}{dt} = \begin{cases} \gamma - r(Z(t)) & \text{if } 0 < Y(t) < M \\ -\{r(Z(t)) - \gamma\}^+ & \text{if } Y(t) = M \\ 0 & \text{if } Y(t) = 0\ . \end{cases} \quad (53)$$

From Equation (51),

$$\begin{aligned} W(t) > M &\Rightarrow X(t) > 0 \text{ and } Y(t) = 0 \\ 0 < W(t) \leq M &\Rightarrow X(t) = 0 \text{ and } 0 < Y(t) < M \\ W(t) = 0 &\Rightarrow X(t) = 0 \text{ and } Y(t) = M\ . \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{dW(t)}{dt} &= \frac{dX(t)}{dt} - \frac{dY(t)}{dt} \\ &= \begin{cases} r(Z(t)) - \gamma & \text{if } X(t) > 0 \text{ and } Y(t) = 0 \\ r(Z(t)) - \gamma & \text{if } X(t) = 0 \text{ and } 0 < Y(t) < M \\ \{r(Z(t)) - \gamma\}^+ & \text{if } X(t) = 0 \text{ and } Y(t) = M \end{cases} \\ &= \begin{cases} r(Z(t)) - \gamma & \text{if } W(t) > 0 \\ \{r(Z(t)) - \gamma\}^+ & \text{if } W(t) = 0\ . \end{cases} \end{aligned} \quad (54)$$

Thus the dynamics of the $W(t)$ process are identical to those of the buffer-content process of an infinite-sized buffer with output capacity γ and input rate $r(Z(t))$ at time t. Therefore to obtain the properties of the $W(t)$ process, for example its probability distribution, all one needs to do is look up the vast literature on the buffer-content process of an infinite sized buffer with output capacity γ and input rate $r(Z(t))$. The structure of the $\{W(t), t \geq 0\}$ process is exploited in the analysis that follow.

Sample paths of $Z(t)$, $X(t)$, $Y(t)$, and $W(t)$ are shown in Figure 12. Define the first passage time V (see Figure 12) as

$$V = \inf\{t > 0 : W(t) = 0 | W(0) = 0, W(0+) > 0\}. \quad (55)$$

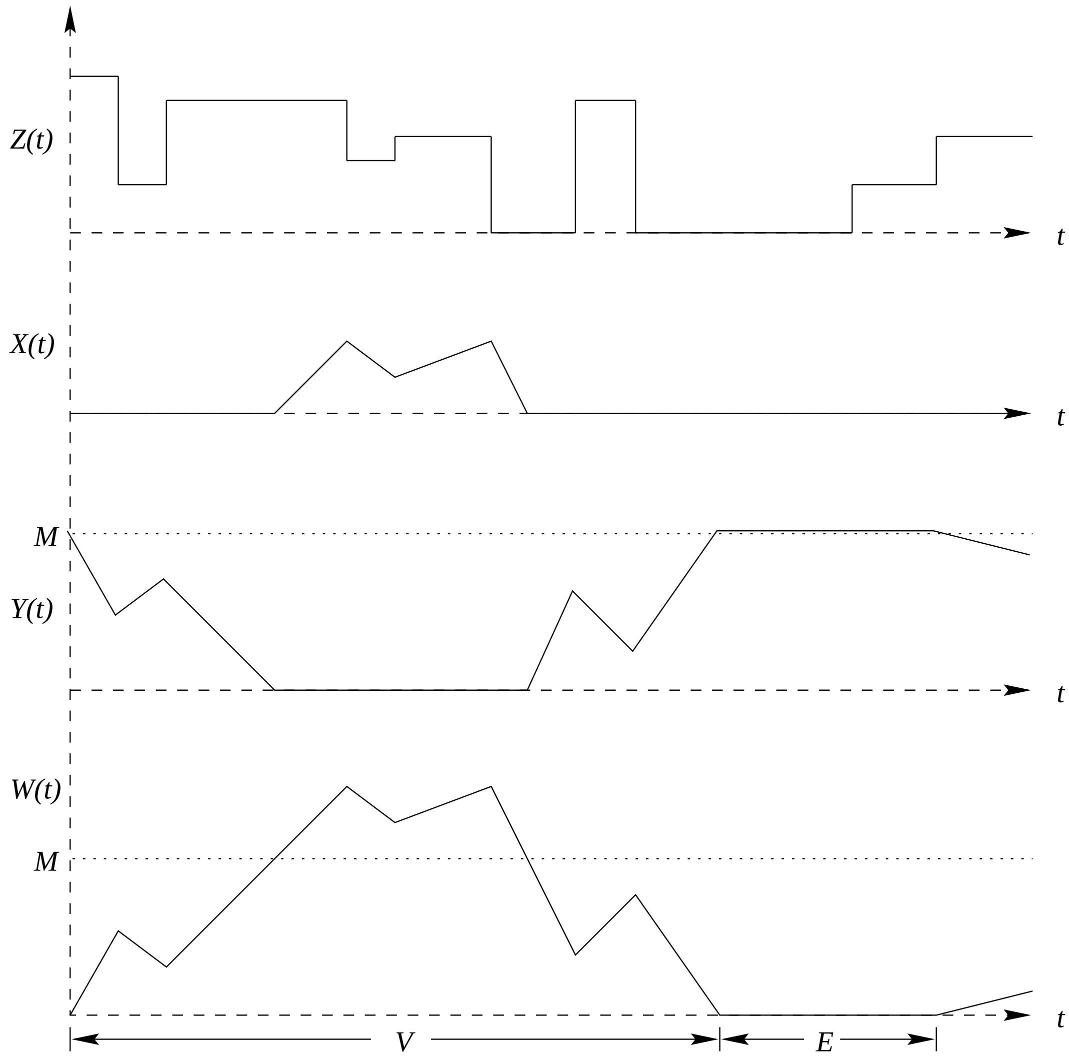

Figure 12: $Z(t)$, $X(t)$, $Y(t)$, and $W(t)$ for Buffered Leaky Buckets

Let $\Theta(V)$ be the total amount of traffic output from the leaky bucket in time V. During the time interval $(0, V)$, $W(t) > 0$ and token pool is non-full. Hence the tokens enter the token pool at rate γ during the time interval $(0, V)$. Since the token pool is full at times 0 and V, the total number of tokens removed from the pool over $(0, V)$ must be the same as the total number of tokens that entered the pool over $(0, V)$. Hence

$$\Theta(V) = \gamma V. \tag{56}$$

Define $A(t)$ as the total fluid arrival into the leaky bucket from the source in time t. Also, let $O(t)$ be the total fluid output from the leaky bucket in time t. Using the result in Equation (56), the following result states the effective bandwidth of the output of the leaky bucket when $\{Z(t), t \geq 0\}$ is a semi-Markov process (SMP). Note that de Veciana [23] derives a similar expression for the effective bandwidth of the output of the leaky bucket for a discrete traffic model.

Let $\{Z(t), t \geq 0\}$ be an SMP on a finite state space $\mathcal{S}$. Let $O(t)$ be the total output from the leaky bucket over $[0, t]$. The effective bandwidth of the output of the leaky bucket

$$eb_O(v) = \lim_{t\to\infty} \frac{1}{vt} \log E\{\exp(vO(t))\}$$

is given in terms of the effective bandwidth of the input, $eb_A(v)$, as

$$eb_O(v) = \begin{cases} eb_A(v) & \text{if } 0 \leq v \leq v^* \\ \frac{v^*}{v} eb_A(v^*) - \gamma \frac{v^*}{v} + \gamma & \text{if } v > v^*, \end{cases} \tag{57}$$

where v^* is obtained by solving

$$\frac{d}{dv^*} \left[v^* eb_A(v^*)\right] = 0$$

and

$$eb_A(v) = \lim_{t\to\infty} \frac{1}{vt} \log E\{\exp(v \int_0^t r(Z(t))dt)\}.$$

For a proof of the above result refer to Gautam [32].

Therefore, given the effective bandwidth of the input traffic to the leaky bucket, it is easy to obtain the effective bandwidth of the output traffic from the leaky bucket by simply replacing the leaky bucket by a single infinite capacity buffer with capacity γ and measuring the output effective bandwidth of this infinite capacity buffer in terms of its input. When the environmental processes of the input traffic can be modeled as Continuous time Markov Chains, Semi-Markov Processes, Markov Regenerative Processes (MRGP) or regenerative processes, etc, one can compute their effective bandwidths using the results shown in Elwalid and Mitra [27], Kesidis et al [47], Kulkarni [50], etc.

5.3.3 Unbuffered Leaky buckets

For the unbuffered leaky bucket, consider only the case when the environmental process governing the fluid input from a source, $\{Z(t), t \geq 0\}$, is a 2-state on-off process ($Z(t) = 0$ or 1, which implies whether the source is off or on respectively at time t). Therefore the fluid input is from a general on-off source with on time distribution $U(\cdot)$ (with mean τ_U) and off time distribution $D(\cdot)$ (with mean τ_D). When the source is on it generates traffic at rate r and at rate 0 when off. Therefore $r(Z(t)) = r\ Z(t)$.

In this unbuffered leaky bucket case, a packet that arrives at the leaky bucket is sent into the network with a "violation" tag if no tokens are available at the time of its arrival.

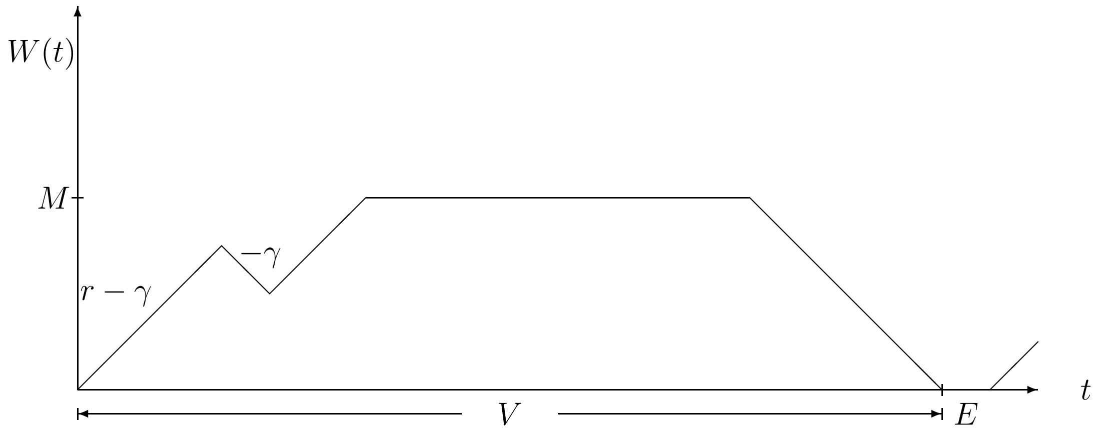

Figure 13: $W(t)$ process for unbuffered leaky bucket

The emphasis will be on the untagged packets as the tagged ones would be dropped in the event of a congestion. Note that $X(t) = 0$ for all t in this unbuffered leaky bucket case. The sample path of $W(t)$ is shown in Figure 13. Since there is no data buffer, $W(t) = M - Y(t)$ and $W(t)$ ranges from 0 to M. Note that $W(t)$ process is identical to a buffer content process of a fluid queue with on-off input, constant output with rate γ, and, a finite buffer of size M.

To obtain the effective bandwidth of the output process, the output rate from the leaky bucket is $R(t)$ at time t and is given by

$$R(t) = \begin{cases} \gamma & \text{if } W(t) = M \\ r & \text{if } W(t) < M \text{ and } Z(t) = 1 \\ 0 & \text{if } W(t) < M \text{ and } Z(t) = 0. \end{cases} \tag{58}$$

Let V be as in Equation (55). Then Equation (56) remains valid in the unbuffered case. Hence the effective bandwidth of the output process from the unbuffered leaky bucket is equivalent to that of of the output process from a single finite buffer (of size M) with general on-off source input and output capacity γ. However, the effective bandwidth of the output cannot be easily written in terms of that of the input due to the fluid loss (as a result of untagged traffic) at the input buffer.

Closed-form algebraic expressions for $eb_O(v)$ are intractable even when the sources are exponential on-off sources. For general on-off sources, an approximation is developed in Gautam [32].

5.4 Wireless Network Models

One of hottest research topics in telecommunications is wireless communications technology and a survey paper would certainly be incomplete without describing some of the on-going research work in mobile communications. However, the field is relatively new and most of the techniques are not well-established. Therefore only a brief summary of some of the current papers in the area of stochastic models in wireless networks are presented here. Almost all the forementioned traffic models, performance analysis, flow control, congestion control, etc do not make any assumptions about whether the networks are at least partially wireless or not. It is to be noted that mobile communications where the users (sources and destinations) are mobile are called wireless communication here. Since the sources and destinations are not static an important problem is to locate the users to send and receive messages.

A theoretical framework for the study of mobility tracking based on user (or for that matter service or host) location probability distributions are provided in Rose and Yates [71]. Using stochastic ordering and information theory, quantitative comparisons of various mobility schemes are demonstrated and insights are obtained into the mobility tracking problem over a wide range of mobility characteristics.

Awduche et al [6] describe location management issues that involve tracking components that maintain dynamic data on the locations of mobile stations through a distributed database. The main focus is on a search component that prescribes the manner in which the wireless network is to be paged so as to determine the location of mobile stations whose whereabouts are unknown. The methods used are based on search theory where a stochastic sequential framework that systematically determines the location of mobile stations situated within a group of cells. Search algorithms are hence developed.

A Poisson-arrival location model (PALM) was introduced in Massey and Whitt [61] in which customers arrive according to a nonhomogeneous Poisson process and move independently through a general location state space according to a location stochastic process. That was extended to a version of PALM to study communicating mobiles on a highway. Leung et al [56] stress the need for combining teletraffic theory and vehicular traffic theory. Their numerical results indicate that both the time-dependent behavior and the mobility of vehicles play important roles in determining the system performance.

5.5 Other Topics

There have been several important topics that have been left out of this exposition. Some of the topics are listed below:

- One of the most critical factor that will enable QoS provisioning in high-speed networks is pricing. F.P. Kelly and colleagues have developed some optimal pricing models (see Kelly [46] for an example).

- ATM switch design and router design involve significant amount of stochastic modeling, particularly queueing. All the multiclass scheduling policies (polling, static priority, waited fair queueing, etc) can be implemented on the currently available switches and routers.

- All the models considered here were unicast where traffic flows from a single source to a single destination. There are interesting stochastic models for multicasting (single source and a few destinations like an Internet classroom with students globally located) and for broadcasting (single source and all nodes as destinations) applications.

- Congestion control aspects at the packet level have not been addressed. The most common scheme is to have small buffers and if the buffer overflows, newly arriving packets are dropped. Modifications of that model include dropping from the beginning of the queue and RED (Random Early Detection: where packets from a non-full buffer are dropped with probability $p(n)$ if there are n packets in the buffer).

- Several scenarios in telecommunication networks (such as client-server systems) can be modeled as Queueing Networks. Walrand [76] provides several applications of Queueing Networks in Telecommunications.

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References

[1] V. Anantharam and T. Konstantopoulos. *An optimal flow control scheme that regulates the burstiness of traffic subject to delay constraints.* Proceedings of the 32nd IEEE Conference on Decision and Control, 3606–3610, vol 4, 1993.

[2] V. Anantharam and T. Konstantopoulos. *Optimality and Interchangeability of Leaky Buckets.* 32nd Allerton Conference, Monticello, IL, 235–244, 1994.

[3] V. Anantharam and T. Konstantopoulos. *A Methodology for the Design of Optimal Traffic Shapers in Communication Networks.* IEEE Trans. Aut. Control, Vol. 44–3, 583–586, 1999.

[4] D. Anick, D. Mitra and M. M. Sondhi. *Stochastic Theory of a Data Handling System with Multiple Sources.* Bell System Tech. Journal, 61, 1871–1894, 1982.

[5] G. Apostolopoulos, R. Guerin, S. Kamat, A. Orda and S.K.Tripathi. *Intradomain QoS routing in IP networks: a feasibility and cost/benefit analysis.* IEEE Network, Vol. 3, No. 5, 42–54, Sept.-Oct. 1999.

[6] D.O. Awduche, A. Ganz and A. Gaylord. *An Optimal Search Strategy for Mobile Stations in Wireless Networks.* 5th IEEE Intl. Conf. on Universal Personal Communications, 1996.

[7] F. Baccelli and R. Bonald. *On the stable behaviors of window flow control.* Proceedings of the 36th IEEE Conference on Decision and Control, 1997.

[8] A.W. Berger and W. Whitt. *Effective Bandwidths with Priorities.* IEEE/ACM Trans. on Networking, Vol. 6, No. 4, Aug., 1998.

[9] A.W. Berger and W. Whitt. *Extending the Effective Bandwidth Concept to Network with Priority Classes.* IEEE Communications Magazine, Aug., 1998.

[10] D.P. Bertsekas and R.G. Gallager. *Data Networks.* 2nd ed. Englewood Cliffs, N.J., Prentice Hall, 1992.

[11] M. Butto, E. Cavallero and A. Tonietti. *Effectiveness of the Leaky Bucket Policing Mechanism in ATM Networks.* IEEE J. Sel. Areas Commun., SAC-9, 335–342, 1991.

[12] F. Callegati, G. Corazza and C. Raffaelli. *On the Dimensioning of the Leaky Bucket policing Mechanism for Multiplexer Congestion Avoidance.* IEEE international conf. on Information Engg., vol.2, 617–21, 1993.

[13] H. Chen and D. D. Yao. *A Fluid Model for Systems With Random Disruptions.* Operations Research 40, No. Suppl. 2, S239–S247, 1992.

[14] H. Chen and D. D. Yao. *Dynamic Scheduling of a Multiclass Fluid Network.* Operations Research 41, No. 6, 1104–1115, 1993.

[15] C. S. Chang and J. A. Thomas. *Effective Bandwidth in High-speed Digital Networks.* IEEE Journal on Selected areas in Communications, 13(6), 1091–1100, 1995.

[16] C. S. Chang and T. Zajik. *Effective Bandwidths of Departure Processes from Queues with Time Varying Capacities.* INFOCOM'95, 1001–1009.

[17] H. Chen and A. Mandelbaum. *Discrete Flow Networks: Bottleneck Analysis and Fluid Approximations.* Math. Opns. Res., 16(2), 408-446, 1991.

[18] K. Chae and A.A. Nilsson. *Performance analysis of prioritized token ring.* Proc. of the 34th Midwest Symposium on Circuits and Systems, 447–450, vol. 1, 1991.

[19] I. Cidon and I.S. Gopal. *Paris : An Approach to Integrated High-Speed Private Networks* Int. J. of Digital and Analog Cabled Systems, Vol. 1, No. 2, 1998.

[20] E. Cinlar. *Introduction to Stochastic Processes.* Prentice-Hall, Engelwood Cliffs, N.J., 1975.

[21] A. Dasylva and R. Srikant. *Bounds on the performance of admission control and routing policies for general topology networks with multiple call classes.* IEEE INFOCOM-99, 505–512, March 1999.

[22] G. de Veciana, C. Courcoubetis and J. Walrand. *Decoupling Bandwidths for Networks: A Decomposition Approach to Resource Management.* INFOCOM'94, 466–473.

[23] G. de Veciana. *Leaky Buckets and Optimal Self-Tuning Rate Control.* GLOBECOM'94, 1207–1211, 1994.

[24] G. de Veciana, G. Kesidis and J. Walrand. *Resource Management in Wide-Area ATM Networks Using Effective Bandwidths.* IEEE Journal on Selected Areas in Communications, 13(6), 1081–1090, 1995.

[25] G. de Veciana and G. Kesidis. *Bandwidth allocation for multiple qualities of service using generalized processor sharing.* IEEE GLOBECOM–94, 1550–1554, 1994.

[26] A.I. Elwalid and D. Mitra. *Analysis and Design of Rate-Based Congestion Control of High Speed Networks, part I: Stochastic Fluid Models, Access Regulation.* Queueing Systems, Theory and Applications, Vol.9, 29–64, 1991.

[27] A.I. Elwalid and D. Mitra. *Effective Bandwidth of General Markovian Traffic Sources and Admission Control of High-speed Networks.* IEEE/ACM Transactions on Networking, 1(3), 329–343, June 1993.

[28] A.I. Elwalid and D. Mitra. *Analysis, Approximations and Admission Control of a Multi-service Multiplexing System with Priorities.* INFOCOM'95, 463–472, 1995.

[29] A.I. Elwalid and D. Mitra. *Traffic shaping at a network node: theory, optimum design, admission control.* INFOCOM'97, 444–454, 1997.

[30] A.I. Elwalid, D. Heyman, T.V. Lakshman, D. Mitra, and A. Weiss. *Fundamental Bounds and Approximations for ATM Multiplexers with Applications to Video Teleconferencing.* IEEE Journal on Selected areas in Communications, 13(6), 1004–1016, 1995.

[31] N. Gautam, V. G. Kulkarni, Z. Palmowski and T. Rolski. *Bounds for Fluid Models Driven by Semi-Markov Inputs.* Probability in the Engineering and Informational Sciences, 13, 429–475, 1999.

[32] N. Gautam. *Buffered and Unbuffered Leaky Bucket Policing : Guaranteeing QoS, Design and Admission Control.* Telecommunication Systems, Vol. 21, No. 1, 35-63, 2002.

[33] N. Gautam. *Effective Bandwidth Methodologies for Network Management.* Master of Science Expository Paper, Dept of Operations Research, University of North Carolina, Chapel Hill, NC 27599, 1995.

[34] R. J. Gibbens and P.J. Hunt. *Effective bandwidths for the Multi-type UAS Channel.* Queueing Systems, Theory and Applications, 9, 17–28, 1991.

[35] R. J. Gibbens, F.P. Kelly and P.B. Key. *Dynamic Alternative Routing – Modelling and behaviour.* Proc. 12th Int. Teletraffic Congr., Turin, Italy, 1988.

[36] R. J. Gibbens, F.P. Kelly and S.R.E. Turner. *Dynamic Routing in Multiparented Network.* IEEE/ACM Trans. on Networking, Vol. 1, No. 2, 261–270, Apr. 1993.

[37] R. J. Gibbens and F.P. Kelly. *Network Programming Methods for Loss Networks.* IEEE, JSAC, Vol. 13, no. 7, 1189–1198, 1995.

[38] S. Gorinsky, S. Baruah and A. Stoyen. *Boosting the network performance via traffic reshaping* Proceedings of the Sixth International Conference on Computer Communications and Networks, 285–290, 1997.

[39] Gu, X., K. Sohraby and D. R. Vaman. *Control and performance in packet, circuit, and ATM networks.* Kluwer Academic, Boston, 1995.

[40] L. Gün, V.G. Kulkarni and A. Narayanan. *Bandwidth Allocation and Access Control in High-speed Networks.* Annals of Operations Research, 49, 161–183, 1994.

[41] J. M. Harrison. *Brownian Motion and Stochastic Flow Systems.* Wiley, 1985.

[42] D. P. Heyman and M. J. Sobel. *Stochastic Models in Operations Research, Vol 1,* McGraw Hill, N.Y., 1982.

[43] D. Holtsinger and H. Perros. *Performance Analysis of Leaky Bucket Policing Mechanisms.* Proc. Tricomm '92, Raleigh, NC, 1992.

[44] Van Jacobson. *Congestion Avoidance and Control.* ACM SIGCOMM–88, Stanford, CA, 1988.

[45] Van Jacobson. *Berkeley TCP evolution from 4.3-tahoe to 4.3-reno.* 18th IETF, Vancouver, BC, 1990.

[46] F. P. Kelly. *Charging and Accounting for Bursty Connections.* Internet Economics, MIT press, 1996.

[47] G. Kesidis, J. Walrand, and C.S. Chang. *Effective bandwidths for Multiclass Markov Fluids and Other ATM sources.* IEEE/ACM Transactions on Networking, 1(4), 424–428, 1993.

[48] T. Konstantopoulos and S. Lin. *High variability versus long-range dependence for network performance.* Proceedings of the 35th IEEE Conference on Decision and Control, Kobe, Japan, 1354–1359, 1996.

[49] K.R. Krishnan, A.L. Neidhardt and A. Erramilli. *Scaling Analysis in Traffic Management of Self-Similar Processes.* Proc. 15th Intl. Teletraffic Congress, Washington, D.C., 1087–1096, 1997.

[50] V. G. Kulkarni. *Effective Bandwidths for Markov Regenerative Sources.* Queueing Systems, Theory and Applications, 24, 1997.

[51] V. G. Kulkarni. *Modeling and Analysis of Stochastic Systems.* Texts in Statistical Science Series. Chapman and Hall, Ltd., London, 1995.

[52] V. G. Kulkarni, and N. Gautam. *Admission Control of Multi-Class Traffic with Service Priorities in High-Speed Networks.* Queueing Systems, Theory and Applications, 27, 79–97, 1997.

[53] V. G. Kulkarni, and N. Gautam. *Leaky Buckets : Sizing and Admission Control.* Proceedings of the 35th IEEE Conference on Decision and Control, Kobe, Japan, 1996.

[54] V. G. Kulkarni, and T. Rolski. *Fluid Model Driven by an Ornstein-Ühlenbeck Process.* Probability in Engineering and Information Sciences, 8, 403–417, 1994.

[55] A. Kumar. *Comparative Performance Analysis of Versions of TCP in a Local Network with a Lossy Link.* IEEE/ACM Trans. on Networking, Vol. 6, No. 4, 485–498, August 1998.

[56] K.K. Leung, W.A. Massey and W. Whitt. *Traffic Models for Wireless Communication Networks.* INFOCOM-94, 1029–1037, 1994.

[57] D.M. Lucantoni, K.S. Meier-Hellstern, M.F. Neuts. *A single-server queue with server vacations and a class of non-renewal arrival processes.* Adv. Appl. Prob., 22, 676–705, 1990.

[58] D.M. Lucantoni. *New results for the single server queue with a batch Markovian arrival process.* Stoch. Mod., 7, 1–46, 1991.

[59] D.M. Lucantoni, G.L. Choudhury and W. Whitt. *Computing transient distributions in general single-server queues.* GLOBECOM '93, 1045 -1050 vol.2, 1993.

[60] M.A. Marsan, E. de Souza e Silva, R.L. Gigno and M. Meo. *A Markovian Model for TCP over ATM.* Telecommunication Systems, Vol. 12, No. 4, 341-368, 1999.

[61] W.A. Massey and W. Whitt. *Network of Infinite Server Queues with Nonstationary Poisson Input.* Queueing Systems, Theory and Applications, Vol.13, 1993.

[62] A. Misra and T.J. Ott. *The window distribution of idealized TCP congestion avoidance with variable packet loss.* IEEE INFOCOM–99, 1564–1572, March 1999.

[63] A. Narayanan and V. G. Kulkarni. *First Passage Times in Fluid Models with an Application to Two-priority Fluid Systems.* Proceedings of the IEEE International Computer Performance and Dependability Symposium, 1996.

[64] Z. Palmowski and T. Rolski. *The superposition of Alternating On-off Flows and a Fluid Model.* Report no 82, June 1996, Mathematical Institute, Wrocław University, 1996.

[65] T.J. Ott and J.G. Shanthikumar. *Discrete Storage Processes and Their Poisson Flow and Fluid Flow Approximations.* Queueing Systems, Theory and Applications, 101–136, 24, 1997.

[66] A. K. Parekh and R. G. Gallager. *A Generalized Processor Sharing Approach to Flow Control in Integrated Services Networks: The Single Node Case.* IEEE/ACM Transactions on Networking, 1(3), 344–357, 1993.

[67] A.A. Puhalskii and W. Whitt *Functional Large Deviation Principle For Waiting And Departure Processes.* Probability in the Engineering and Informational Sciences, 12, 479–507, 1999.

[68] L.R. Rabiner. *A Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition.* Proceedings of the IEEE, Vol 77, No. 2, Feb. 1989.

[69] A. Raha, S. Kamat and W. Zhao. *Using traffic regulation to meet end-to-end deadlines in ATM LANs.* Proceedings of the International Conference on Network Protocols, 152–159, 1995.

[70] A. Romanow and S. Floyd. *Dynamics of TCP traffic over ATM Networks.* IEEE JSAC, vol. 13, no. 4, 633–641, May 1995.

[71] C. Rose and R. Yates. *Location Uncertainty in Mobile Networks: A Theoretical Framework.* IEEE Communications Magazine, vol. 35, no. 2, 94–101, Feb. 1997.

[72] R.R. Roy. *Continuous Time Markov Chain Model for Token Ring Local Area Network.* IEEE Aerospace Applications Conference, 247–251, 1990.

[73] K. Sohraby and M. Sidi. *On the Performance of Bursty and Modulated Sources Subject to Leaky Bucket Rate-based Access Control Schemes.* IEEE Trans. on Commun., Vol. 42, No. 2/3/4, 1994.

[74] W. Stevens. *TCP/IP Illustrated.* Vol 1 and 2, Addison-Wesley, 1994.

[75] S. Vamvakos and V. Anantharam. *On the departure process of a leaky bucket system with long-range dependent input traffic.* Queueing Systems, Theory and Applications, Vol. 28, 1–3, 1998.

[76] J. Walrand (1988). *An Introduction to Queueing Networks.* Prentice hall, Englewood Cliffs, NJ.

[77] J. Walrand and P.P. Varaiya. *High-Performance Communication Networks.* Morgan Kaufmann Publishers, Inc., 1996.

[78] W. Willinger and V. Paxson. *Where Mathematics meets the Internet.* Notices of the American Mathematical Society, 45(8), pp. 961–970, Sept. 1998.

[79] W. Willinger, M.S. Taqqu and A. Erramilli. *A bibliographical guide to self-similar traffic and performance modeling for modern high-speed networks.* Stochastic Networks: Theory and Applications, F.P. Kelly, S. Zachary and I. Ziedins (Eds.), Royal Statistical Lecture Note Series, Vol. 4, pp. 339–366, Clarendon Press, Oxford, UK, 1996.

[80] G. Wu and J.W. Mark. *Discrete Time Analysis of Leaky Bucket Congestion Control.* Proc. of ICC '92, 1992.

[81] N. Yin and M.G. Hluckyj. *Analysis of the Leaky Bucket Algorithm for on-off Data Sources.* Journal of High Speed Networks, Vol. 2, no.1, 81–98, 1993.