

# **A Mathematical Programming Model for Scheduling Pharmaceutical Sales Representatives**

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## **Abstract**

To increase revenues, pharmaceutical companies rely on their sales forces to promote new and existing drugs to physicians. Compensation for sales representatives is largely commission based. However, individual effort may not be the driving force behind compensation. Sales representative might be unfairly rewarded only because they are assigned to physicians with high sales potential. Given a set of physicians and representatives, a mathematical program is developed to maximize profit for the pharmaceutical company, while balancing both workload and sales opportunities for representatives. The model is valuable for daily operational decisions. The methodology applies to any multi-product sales-force based industry.

## **Keywords**

Sales force assignments, mathematical programming, design of experiments

## **1. Introduction**

Pharmaceutical sales companies are vigilant in their efforts to raise profits by increasing the sales force size and the number of visits to physicians. Scott-Levin Associates [1] found that 1995 to 1999 the number of pharmaceutical sales representatives increased 61% to nearly 80,000 from 50,000, and that visits by sales representatives to doctors' offices increased ten percent to 36 million from 33 million. There is a great potential for pharmaceutical companies to turn a significant profit by employing a strong sales force. However, an efficient use of this sales force is required to achieve increased revenues. Of concern is that sales representatives are often rewarded for their efforts through commissioned based salary packages. Companies may unintentionally reward their employees, not solely on their efforts, but as a function of the physicians to which they are selling. A sales representative may have higher sales due to the nature of the assigned physician or territory. Furthermore, daily decisions regarding which physician each sales representative should visit are determined by gut feel and intuition, as opposed to a scientific mechanism.

To address these shortcomings, an optimization problem is developed in this paper. The profiles of physicians and sales representatives, which specify the geographic location of each individual, the drugs that are either prescribed or pitched, and the probability that these drugs will be sold are given. The aim is to determine a schedule for which pharmaceutical sales representatives will visit which doctors for the daily operational decisions. The objective is to maximize the expected profit for the pharmaceutical sales companies, while constraining the assignments to ensure an equal distribution of workload, distances traveled, and opportunity for sales across all representatives. Also considered are the different solutions that occur when varying the profiles of physicians and sales representatives. A primary benefit of the approach is that the sales manager does not have to be an expert in mathematics to interpret the solutions generated. Also, sales representative to physician assignments can be executed on a daily basis due to the simplicity of the formulation and the level of computational complexity. Finally, the playing field for the sales force is leveled, and a cost avoidance for pharmaceutical companies is achieved by a reduction in bonus inflation.

This paper is organized as follows. In the next section, we present some background material relevant to this research. Then, in Section 3, we describe our mathematical programming formulation. We present numerical results for several problem instances in Section 4. We present concluding remarks in Section 5.

## **2. Background**

For multi-product industries (such as pharmaceuticals), the number of sales forces and the products assigned to each sales force are of concern. Rangaswamy et al [2] discuss this issue extensively and develop a model to handle these concerns. These complex decisions are not the operational, day-to-day decisions that are the focus of this paper. Yet, the model is useful in that it helps to determine some of the strategic and tactical decisions. This information indicates that the assignment of drugs to sales representatives can be viewed as a separate problem. Accordingly, the assumption made in this paper is that each sales representative's drug assignments will be predetermined.

Sales force sizing, salesperson location, and territory alignment are assumed to be solved and given for the work of this paper. These issues are thoroughly explored by Drexel and Haase [3]. The problem of allocating the sales force (i.e. which sales representative will visit which physician) is of particular interest since it relates to the work done in this paper. Drexel and Haase [3] use a nonlinear mixed-integer programming model for this problem, however they do not take into account the workload balance issues for the sales representatives. In addition, the decision maker (the sales force manager) is far removed from the process; it would be difficult for a meaningful solution to be achieved. The formulation in this paper is simplistic in nature and can be suitably modified by the decision maker.

The necessity for model solutions to have practical meaning for the decision maker is one of a number of insights provided in Sinha and Zoltners [4], which presents insights from over 25 years of real world experience. An interesting illustration is that throughout their collective careers, the authors have conducted over 2000 real world studies related to sales force management; yet, within all studies, the decision maker has never been the model user. It is their perspective that this is the favorable approach to solving the sales force problems. Perhaps a better approach is to develop a simple model that enables the decision maker to proactively solve his or her own sales force problems independently, without the aid of an outside source. It is with this approach that a truly successful model is developed in that both the methodology and the solution itself are financially beneficial to the company.

### 3. Formulation

#### 3.1 Objective and Requirements

In a generalized pharmaceutical sales problem, on a given day there is a set of “ $J$ ” physicians to be visited. These physicians prescribe various types of drugs and a set of “ $I$ ” sales representatives trained to promote various combinations of drugs. The sales potential is the expected profit made by the pharmaceutical company by the sales rep to physician visit. Therefore, it is a measure of the probability that a physician will prescribe the drug that the sales representative is promoting. The relationship between each drug and each physician is rated on a scale of 100 by considering the characteristics of the drug, the type of physician that is in question, and the historical performance of the physician. In this formulation, “ $P_{ij}$ ” will represent a sum of the potential that exists when sales representative  $i \in (1, 2, \dots, I)$  promotes the drugs they are trained on to physician  $j \in (1, 2, \dots, J)$ . The goal is to develop a model in which sales reps are assigned to physicians in such a way that total sales potential over all sales representatives and physicians is maximized for the pharmaceutical company.

An important constraint to the model is that the sales potential must be approximately equal for all representatives. However, there will be some slight variation among the sales representatives to obtain feasible solutions to the problem. Similarly, the objective of maximizing the potential is also constrained by the fact that the workload for the different sales representatives must be relatively equal. This is an important constraint to create a sense of fairness in the selling environment. When comparing the sales representatives to one another, the differences in the number of physicians visited between reps should be close to zero. Another requirement that only one sales representative can visit each physician on a given day. Finally, the distance that each sales representative can travel must be limited by an upper bound (the maximum distance is denoted as  $MD$ ).

#### 3.2 Approximation for the Distance Constraint

All the constraints in the previous subsection, except the final one, can be handled in a straightforward manner. One of the key ideas in this paper is an engineering approximation to tackle the distance constraint. Each rep starts from their home base, visits the set of physicians and returns to their home base at the end of a day. Now consider the case where each rep returns to their home base after visiting each physician. We approximate this distance to be about twice the distance the rep actually travels. Therefore, the actual distance is approximated as the average sum of one way distances from the rep’s home base to each physician, multiplied by the average physician visits plus one (for the return trip home), multiplied by a safety factor (initially assumed to be 1.25). This is used to solve the mathematical program. Then for each rep we solve a traveling salesperson problem (or TSP; see Ahuja et al [5]) to determine the order of physicians to visit. Then if the total distance traveled by every rep is less than  $MD$ , we are done. Otherwise, we change the factor, and iterate between the mathematical program and the TSP. In most realistic cases, the constraint is not binding and a single iteration is sufficient.

### 3.3 Solution Methodology

Some variables are defined in Subsection 3.1. If sales rep  $i$  is assigned to physician  $j$ , let the decision variable  $X_{ij} = 1$ , otherwise  $X_{ij} = 0$ . Next, when comparing two sales reps, say  $i$  and  $k$  (where  $k \in \{1, 2, \dots, I-1\}$ ) slight deviations are allowed in the balance of the workload. Let the workload deviations be represented by the negative and positive slack variables,  $W_{ik}^-$  and  $W_{ik}^+$ , respectively. These workload slack variables are bounded by lower and upper limits,  $WL$  and  $WU$ , respectively. Similarly, let  $P_{ik}^-$  and  $P_{ik}^+$  represent the negative and positive slack variables for the sales potential deviations between sales reps  $i$  and  $k$ . Let  $PL$  and  $PU$  be the lower and upper bounds for the negative and positive potential slack variables, respectively. Finally, let  $D_{ij}$  be the scaled distance between home base of sales rep  $i$  and physician  $j$ ; and, the maximum sum of one-way distances for the rep cannot be greater than  $MD$ .

Considering the objective and the constraints, the mathematical programming model can be stated as:

$$\text{Maximize:} \quad Z = \sum_{j=1}^J \sum_{i=1}^I P_{ij} X_{ij} \quad (1)$$

$$\text{Subject to:} \quad \sum_{i=1}^I X_{ij} = 1 \quad \text{for all } j \in [1, J]; \quad (2)$$

$$\sum_{j=1}^J X_{ij} - \sum_{j=1}^J X_{kj} + W_{ik}^- - W_{ik}^+ = 0 \quad \text{for all } k = 1, \dots, i-1; \quad i \in [1, I]; \quad (3)$$

$$W_{ik}^- \leq WL \quad \text{for all } k = 1, \dots, i-1; \quad i \in [1, I]; \quad (4)$$

$$W_{ik}^+ \leq WU \quad \text{for all } k = 1, \dots, i-1; \quad i \in [1, I]; \quad (5)$$

$$\sum_{j=1}^J P_{ij} X_{ij} - \sum_{j=1}^J P_{kj} X_{kj} + P_{ik}^- - P_{ik}^+ = 0 \quad \text{for all } k = 1, \dots, i-1; \quad i \in [1, I]; \quad (6)$$

$$P_{ik}^- \leq PL \quad \text{for all } k = 1, \dots, i-1; \quad i \in [1, I]; \quad (7)$$

$$P_{ik}^+ \leq PU \quad \text{for all } k = 1, \dots, i-1; \quad i \in [1, I]; \quad (8)$$

$$\sum_j D_{ij} X_{ij} < MD \quad \text{for all } i \in [1, I]; \quad (9)$$

$$X_{ij} \geq 0 \quad \text{for all } i \in [1, I]; \quad j \in [1, J] \quad (10)$$

The objective function (1) represents the expected potential sales procured with the visit of each sales representative to their corresponding physician. Since the model is to be used to determine the daily schedule of sales reps to physicians, it is required that only one rep visit each physician per day, forced by constraint (2). Constraint (3) indicates that the difference in the number of physicians that each sales representative is assigned to should be as close to zero as possible, allowing some deviation. This deviation in workload is controlled by constraints (4) and (5). The lower and upper limits of these constraints are specified by the decision maker and will determine how much deviation is allowed. Similarly, constraint (6) requires that the difference between sales potential of any two reps be as close to zero as possible, with some deviation allowed. Constraints (7) and (8) require that there are limits on the lower and upper deviation, respectively. Again, these lower and upper bounds are at the hands of the decision maker. Constraint (9) is the approximation of the maximum distance that each sales rep is allowed to travel.

Once the reps are assigned to physicians, we solve TSPs to determine the order of physicians to visit for each rep. Since the number of physicians is usually small, the TSPs are solved by complete enumeration. The solution to the TSPs would also give us the actual distance traveled by each rep. If the distances are far greater than  $MD$ , we rescale the distances  $D_{ij}$ , solve the mathematical program and the TSPs iteratively till all distances are less than  $MD$ .

### 3.4 Comparison to the Multi-Depot Vehicle Routing Problem

A similar problem to the one described is known as the multi-depot vehicle routing problem (MDVRP), where in a given geographic region, a set of known customers are supplied by a set of vehicles. The customers and vehicles can be compared to the physicians and sales reps, respectively. Upon examining Renaud et al [6], which further develops the basic assumptions of the MDVRP, a number of differences can be seen. The main objective of the MDVRP is to minimize distance, so the assignments of vehicles to customers (or sales representatives to physicians) are severely limited by location. Most often, a graphical display of the assignments show clusters throughout the

geographic region around the location of each depot (or sales representative location). Conversely, the main objective of this paper is to maximize the sales potential, subject to balancing the working environment. In practice, the limits on the distances that sales reps can travel are significantly relaxed. It is possible that a representative and a physician who are located in essentially the same place will not be assigned to one another. Also, the MDVRP assumes that any sales representative can visit any of the physicians, illustrating a uniform workforce. However, in this application, neither the sales force nor the physicians are homogenized. Finally, it is noted that the MDVRP is an NP-hard problem. The mathematical program developed in this paper solves problems optimally (although the TSP is NP-complete our problems are relatively small).

## 4. Results

### 4.1 Sample Problem

To illustrate the problem, a large geographic region was tested of in a rectangular fashion, 200 miles long by 100 miles wide. Within this region, 20 physicians and 7 sales representatives were placed randomly according to a uniform distribution within this 2-dimensional space. Next, it was decided that a set of six drugs would be used for this sample problem. Of those six drugs, the sales representatives would be able to promote a subset of three drugs. A profile for each physician and sales representative was created randomly to determine which drugs were pitched (for sales reps) or potentially prescribed (for physicians). No two sales representatives promote exactly the same drugs. Next, the distances between each sales representative and each physician were calculated. Then the limits on the slack variables for workload and potential were set at 2 and 150, respectively. The maximum distance for the distance constraint is set at 405 miles. Using the mathematical program, a complete linear program was written and solved. First, assignments of sales representatives to physicians was determined. The computation time to solve the problem was eight seconds. The results of these assignments are presented in Table 1. The workload for each of the sales representatives is balanced with two to four physicians each. The variation is due to the tolerance allowance of the slack variables.

Table 1. Solution to Sample Problem

Sales Rep	Doctors Visited	Distance	Potential
1	20-3-2	390	200
2	13-6	248	130
3	18-10	271	145
4	8-1-12	271	256
5	15-16-7	296	242
6	17-14-11-5	327	293
7	19-9-4	379	238
<b>Total Potential</b>			1504

For the second phase, the order in which physicians are visited was determined using a TSP. The “Doctors Visited” column of Table 1 indicates the order in which the physicians should each be visited.

To analyze these results further, the actual paths that each sales representative travels are examined in Figure 1 below, where R1 to R7 are the seven sales representatives. Most problems dealing with the sales force assignments (and MDVRPs) aim to minimize the total distance traveled. A graphical display of the sales representatives’ paths would be quite clustered; each sales representative would generally visit physicians in close proximity. However, with our objective, these paths are not so clustered. In this example, many of the paths cross over each other. Since the goal was to balance the working environment, sales representatives are required to travel extra distances. So, it is expected that such scattered paths are achieved.

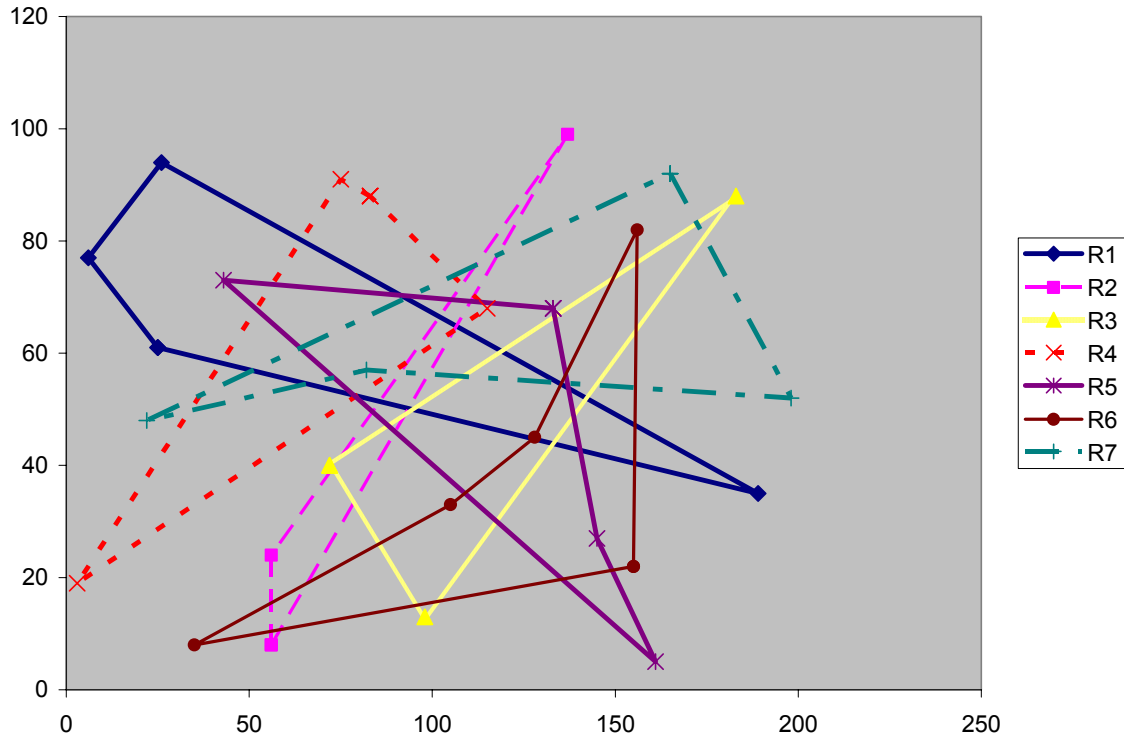


Figure 1. Graphical Solution to Sample Problem

#### 4.2 Designed Experiment

In this sample problem, the physician and sales representative profiles were created to the given specifications. A number of factors in the sample problem can be varied to determine this variation in terms of sales potential and the paths traveled. Hence, a design of experiments approach (DOE) is selected for analysis (see Montgomery [7]). By varying the factors and levels shown in Table 2, there exist 64 possible combinations. The solution times ranged from eight seconds to 30 minutes.

Table 2. Factors and Levels for DOE

Factor	Code	Level	Description
Distance Constraint	D	1	Liberal: sum of distances for each rep $\leq 300$
		2	Stringent: sum of distances for each rep $\leq 200$
Sales Rep-Drug Assignment	U	1	Homogeneous: Each rep can promote the same three drugs
		2	Random: Rep is assigned a subset of 3 out of 6 drugs to promote
Physician Characteristics	X	1	Each doctor prescribes the same 4 drugs; sum potential of 100 for ea
		2	Randomly assigned 4 out of 6 drugs; sum potential of 100 for ea
		3	Each prescribes the same 4 drugs; 100 mean potential across doctors
		4	Randomly assigned 4 out of 6 drugs; 100 mean potential across doctors
Sales Rep Location	Y	1	Spread out evenly
		2	Clustered centrally
Physician Location	Z	1	Spread out evenly
		2	Clustered in four corners

To analyze the effect varying the factors has on potential and to determine the significant and optimal levels of the factors, statistical analysis was conducted. Assuming a confidence level of 99%, significant factors are the distance constraint, sales rep assignments, physician characteristics, and physician coordinates. The only factor that is not significant is the sales representative coordinates. The fact that the sales representative coordinates do not affect the total potential is an interesting result. Different assignments are achieved when using the dispersed locations or the centralized locations. Yet, the constraints of the problem are still satisfied, and the objective function of maximizing

total sales potential is unchanged. The significant factor interactions are DU, DZ, UX, and UZ. Effects plots were used to determine the optimal factor levels. The recommended factor levels are discussed below and are in Table 3.

Table 3. Recommended Levels of Each Factor

Factor	Code	Level	Description
Distance Constraint	D	1	Liberal distance constraint
Sales Rep-Drug Assignment	U	2	Random subset of drugs assigned to sales reps
Physician Characteristics	X	2 or 4	Random subset of drugs prescribed by physician with either mean or sum potential of 100 across physicians
Sales Rep Location	Y	Either	Distributed or centralized sales rep location
Physician Location	Z	1	Distributed physician location

The liberal distance constraint is recommended, a logical result since the looser the constraint, the more flexible the solution. The sales representative assignments (factor U), maximize potential with reps assigned a random subset of the drugs available to sell. This result is expected since a homogenized sales force cannot promote some of the drugs, reducing the possibility for profit. The physician characteristics (factor X) are optimized when all physicians prescribe the same drugs with either a mean or a sum potential across doctors of 100. The sales representative coordinates (factor Y), have a negligible effect on the total potential achieved. This conclusion is drawn from both the main effects plot, and from the high p-value of 0.545 observed in the analysis of variance. The constraints are satisfied while maximizing the objective function regardless of where sales reps are located. This insight is helpful, since sales reps are typically encouraged to live in a centralized location. Finally, the doctor coordinates appear to result in higher overall sales potential when the physicians are spread evenly across the territory. This is an expected trend, since there is greater flexibility offered for assignments of sales representatives when the physicians are not clustered in groups. Additionally, more often than not physicians are spread out across sales territories; it is beneficial that this arrangement reveals an increase in sales potential.

## 5. Concluding Remarks

There are a number of significant contributions achieved by the research presented in this paper. First, by testing the developed model, it is shown feasible solutions can be generated to the sales force assignment problem while taking into account fairness and profit maximization. The ultimate benefit is that pharmaceutical sales companies can then reward its sales representatives bonuses that would be reflective of the effort put forth by the salesperson, as opposed to rewarding them on the sheer luck of their assignments. The approximation to tackle the distance constraint rendered the problem to be easily solvable. In addition, using analysis of variance techniques, it was determined that the physician and sales representative profiles do have an impact on the goal of increasing the total potential for sales. The results of the application showed that the location of the sales representatives did not play a significant role in affecting the potential profit made by the pharmaceutical sales company. This is significant since pharmaceutical companies typically encourage sales representatives to live in a centralized location. Finally, the computational complexity of this model was fairly simple, which is a major contribution for sales managers. The decision maker can control the constraints on the problem and can customize the model to reflect the business's preferences. The model can be used to solve the daily operational decisions of assigning sales representatives to physicians, taking some of the guess work out of operational sales force management.

## References

- [1] Green, Jay, March 27, 2000, Drug reps targeting nonphysicians, *www.amednews.com*
- [2] Rangaswamy, Arvind, Sinha, Prabhakant & Zoltners, Andris, 1990, An integrated model-based approach for sales force structuring. *Marketing Science*, **9** (4), 279-298
- [3] Drexler, Andreas & Haase, Knut, 1999, Fast approximation methods for sales force deployment. *Management Science*, **45** (10), 1307-1323
- [4] Sinha, Prabhakant & Zoltners, Andris, 2001, Sales-Force Decision Models: Insights from 25 Years of Implementation, *INTERFACES*, **31** (3), S8-S44
- [5] Ahuja, Ravindra K., Magnanti, Thomas L., Orlin, James B. (1993). **Network flows: theory, algorithms, and applications.** Upper Saddle River, NJ
- [6] Renaud, Jacques, Laporte, Gilbert, & Boctor, Fayez F., 1996, A tabu search heuristic for the multi-depot vehicle routing problem. *Computers and Operations Research*, **23** (3), 229-235
- [7] Montgomery, Douglas C. (2001). **Design of Analysis and Experiments.** New York, NY.