



PERGAMON

Transportation Research Part B 35 (2001) 237–254

---

---

TRANSPORTATION  
RESEARCH  
PART B

---

---

www.elsevier.com/locate/trb

# Probability of breakdown at freeway merges using Markov chains

Jodie L. Evans \*, Lily Elefteriadou, Natarajan Gautam

*College of Engineering, The Pennsylvania State University, USA*

Received 23 November 1998; received in revised form 20 August 1999; accepted 25 August 1999

---

## Abstract

The objective of this paper is to review our recent research completed on the determination of the probability distribution of the time of breakdown on a freeway. The methodology applied was unique, in that, it applied Markov chains to develop the probability distribution of the time of breakdown.

To develop an improved methodology for the prediction of breakdown, the probability distribution of the time of breakdown was determined based on the zonal merging probabilities with respect to the vehicles traveling on the throughway. Freeway flow, available gaps, and drivers' actions as they approach the merge area were taken into consideration in developing the model.

The analytical model was developed based on the information obtained from the literature and an initial field data sample. After determining the arrival distributions of the merging vehicles and the probability of transitioning from state to state, the probability distribution of the time of breakdown occurring was determined before any time,  $t$ , through a model analysis which applied Markov chains and implemented it in MATLAB® code. Finally, field data were reduced and applied in the model validation.

With respect to the application of Markov chains, it was expected that higher arrival rates of the vehicles traveling on the throughway would lead to higher probabilities of breakdown. This is logical since one would expect a greater probability of breakdown occurring when there are a large number of vehicles entering the system. The model did produce results which predicted a higher probability of breakdown for higher arrival rates.

The significance of the results is a better understanding of the prediction of breakdown and capacity estimation. With an improved concept of breakdown on freeways, better freeway analysis and simulation software may be developed. © 2001 Elsevier Science Ltd. All rights reserved.

*Keywords:* Breakdown; Merge; Markov chains

---

---

\* Corresponding author. Present address: McM/McMahon Associates, 70 Fayette Street, Watertown, MA 02472, USA. Tel.: +1-617-923-9140; fax: +1-617-725-0049.

*E-mail address:* joleath@yahoo.com (J.L. Evans).

## 1. Introduction

In freeway operations, merging sections are one of the greatest areas of conflict, and the most difficult sections of the freeway to analyze satisfactorily (Ostrom et al., 1993). Freeway merges are typically designed to allow high speed merging maneuvers without disrupting main freeway traffic (Highway Capacity Manual, 1994). Merging vehicles try to find gaps in the freeway flow. Higher freeway flow means smaller gaps in the traffic stream, and reduced opportunities for vehicles to merge causing them to stop or force their way into the stream (Elefteriadou, 1994).

Although it is believed that capacity and breakdown are interrelated, this relationship is not well-defined in such documents as the Highway Capacity Manual (HCM). Due to the lack of data at or near the capacity level for the definition of speed–flow–density relationships, the process of flow breakdown in merge areas has not been adequately documented (Elefteriadou, 1994). The existing models predict the breakdown as a deterministic function of a given flow rate, or speed, or density, and these models generally assume that breakdown will occur at capacity flows only. The HCM currently defines breakdown through a deterministic process, which is based on a US averaged value for capacity. It should also be noted that for the same volumes, breakdown does not always occur, which is assumed within the HCM deterministic process.

Recent research has found that capacity cannot always be measured in the field immediately prior to breakdown, but rather capacity may occur without the occurrence of breakdown (Elefteriadou, 1994). A quantitative model describing the process of flow breakdown around entrance ramps was developed by Elefteriadou using general probability relationships, which is very useful for the operational analysis of ramp-freeway junctions and the establishment of freeway control strategies to maximize flow and optimize operations on the freeway (Elefteriadou, 1994).

The model developed in this paper follows Elefteriadou's model in that the freeway flow and the respective gaps available on each freeway lane, as well as the drivers' actions as they approach the merge area, are taken into consideration in developing the model (Elefteriadou, 1994). To develop an improved methodology for the prediction of breakdown, the probability distribution of the time of breakdown based on zonal merging probabilities is the method to be applied. In contrast to most existing merging models, Markov chains rather than computer simulation will be applied.

The objectives for this paper are as follows:

- (a) Complete a thorough review of the literature available on freeway merging models and Markov chain applications in traffic operations;
- (b) Develop an analytical model for the prediction of breakdown on a freeway merge;
- (c) Analyze the model to obtain the frequency distributions of the breakdown;
- (d) Collect accurate and applicable data and validate the model; and
- (e) Propose potential applications of the model, potential extensions, and possible drawbacks.

An analytical model was developed based on the information obtained from the literature and initial data sample. After determining the arrival distributions of the merging vehicles and the probability of transitioning from state to state, the probability distribution of the time of breakdown occurring may be determined before any time,  $t$ , by applying Markov chains.

After developing the model, the field data were reduced and applied in the model validation. It should be noted that Markov chains are appropriate for this application because they are probability models for stochastic processes (Hiller and Lieberman, 1995), of which freeway merging maneuvers are one.

## 2. Literature review

### 2.1. Historical review

A historical review of the literature was completed including an in-depth review of several articles relevant to the topic of discussion in this report. The following paragraphs summarize the unique and common key points discussed in each of the articles reviewed.

The earliest research relevant to this topic was by Haight et al. (1962) whose unique key point was treating a merge as stop controlled/uncontrolled intersection. Their description of merging depending on leading and following vehicles was applied by others in future research.

Other early research completed in the 1960s was by Buhr, Drew, Wattleworth, Williams, and Whitson on various projects (Buhr et al., 1967; Drew et al., 1968). Their unique research points included time orientation techniques, a quantitative method for determining the influence of geometrics on merge capacity/LOS, and the assumption that continual merge vehicles are waiting. Common points applied in their research included study sites operating at or near capacity while unaffected by up/downstream and regression analysis (critical gap and slope of gap acceptance line) performed.

In the 1970s relevant research was completed by Szwed and Smith (1972) and Blumenfeld and Weiss (1979). Unlike their predecessors, Szwed and Smith simulated merging with Monte Carlo techniques to generate probability distributions and applied a Markov renewal process with car-following/non-following groups of states. Also, the latter set of authors examined effects on delay and capacity if there was a consistent critical gap, and the variability in gap acceptance was assumed to be due to the inconsistency of drivers. Common to other authors, Szwed and Smith predicted the probability for a given volume that a driver is unable to merge within a distance.

In the early 1990s, relevant research was completed by Banks (1991), Hall and Agyemang-Duah (1991) and Nihan and Berg (1992). Their unique research points included the following: consideration if ramp metering can increase bottleneck capacity by delay of breakdown, confirmation that flow decreases at breakdown, and found that the two-capacity phenomenon was unlikely to provide a metering basis (Banks); consideration of whether maximum flow rates reduce when a queue forms and consideration of maximum flow distribution (Hall and Agyemang-Duah); evaluation of the feasibility and usefulness of an algorithm for metering that anticipated capacity reduction, attempted to reduce or eliminate breakdown conditions by restricting upstream metering rates, tested on-line, and applied time-series intervention analysis (Nihan and Berg). Common to other researchers, Nihan and Berg applied linear regression analysis.

In recent years, research has been completed by Cassidy and Windover (1995), Elefteriadou et al. (1995), Akcelik (1996), Yagar and Hui (1996), Persaud et al. (1998). Different from others, Cassidy and Windover used cumulative arrival curves and evaluated disturbances by comparing cumulative curves. Likewise, Elefteriadou et al. developed a probabilistic aspect of merge breakdown through examination and analysis of data, estimated the probability distribution of the time of breakdown as a function of clusters, and considered flow, gaps, and driver actions. Akcelik explained density, speed, flow and travel time model relationships for uninterrupted and interrupted flows, and provided formulae, examples, and basic relationship among speed, density,

spacing and flow. Also, Yagar and Hui discussed adding capacity to freeways, determined if/where to restripe to gain capacity, and applied a systems analysis procedure for evaluating broad effects of gains. Finally, Persaud et al. defined breakdown as freeway operation near a bottleneck entrance when changing from free flow to a queue, determined the time of breakdown by visual inspection, measured mean queue breakdown flow over intervals, found mean queue discharge flow by computing mean flow after the transition period until speeds recovered to uncongested levels, and determined the probability distribution of the time of breakdown as instances mean queue breakdown flow fell within a range divided by frequency of flows.

## 2.2. Other relevant research

Although not critically reviewed for this paper, it is important to note other relevant research regarding Markovian applications and freeway driver behavior. Research has been completed applying Markovian principals to transportation, but not to the same extent as presented in this paper, by Makigami et al. (1983) and Davis and Kang (1993). Also, freeway driver behavior research has been completed by several authors, including: Olsen and Hostetter (1976), Michaels and Fazio (1989), Lunenfeld (1993), Hunt and Yousif (1994) and Bunker and Troutbeck (1996).

## 2.3. Review of Markov chains

Because a Discrete Time Markov Chain (DTMC) was applied in this research project, a brief review of DTMC's is included here. A DTMC is formed as a set or collection of random variables  $X_0, X_1, X_2, X_3, \dots, X_n$  (Hiller and Lieberman, 1995). DTMC's follow the Markov Property where the conditional probability of future states of the stochastic process depend on the present state and not on the past.

The conditional probabilities are called transition probabilities. A transition probability is necessary to draw the transition diagram for the DTMC. Properties of the transition probability,  $P$ , are that (i) all entries have values between 0 and 1, and (ii) rows must add up to 1. An example transition probability matrix and transition diagram (Fig. 1) is shown below.

$$\begin{array}{cc}
 & \begin{array}{c} A \quad B \end{array} \\
 \begin{array}{c} P = A \\ B \end{array} & \begin{bmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \end{bmatrix}
 \end{array} \tag{1}$$

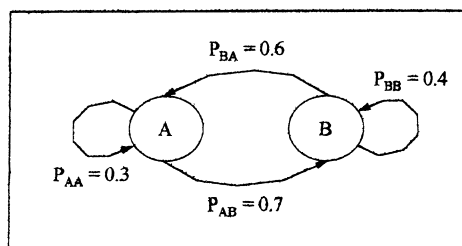


Fig. 1. Example DTMC.

The long-run or steady-state properties of the DTMC for a transition probability with  $i$  rows and  $j$  columns are defined as the following:

$$P_{ij} = \text{transition probability scalar for states } i \text{ by states } j$$

$$\lim_{n \rightarrow \infty} (P_{ij})^{(n)} = \Pi_j \tag{2}$$

where  $\Pi_j$  is the same down any column  $j$ .

$$\Pi_j = \sum_i \Pi_i (P_{ij})^{(1)}, \tag{3}$$

and

$$\sum_i \Pi_i = 1. \tag{4}$$

For a more extensive review of Markov chains, please refer to “An Introduction to probability models” by Ross.

### 3. Model of breakdown

The model of breakdown developed in this paper represents one direction of a four-lane freeway with a single lane on-ramp entering from the right. The area around the merge is divided into zones with approximate length of 175 ft, representative of a typical 2-s time headway between sequential vehicles traveling at 60 mph. The 175-ft length also allowed the initial number of zone–lane pairs to be restricted to a reasonable number. Three zones are defined starting at the nose of the gore point, which leads to two zones in the merge lane and three zones in the through lanes. Therefore, there are eight zone–lane pairs.

The probability distribution of the time of breakdown was determined based on the zonal probabilities of vehicles merging onto a freeway with respect to the other vehicles traveling on the main throughway. Fig. 2 demonstrates an example of the aforementioned zonal probabilities,

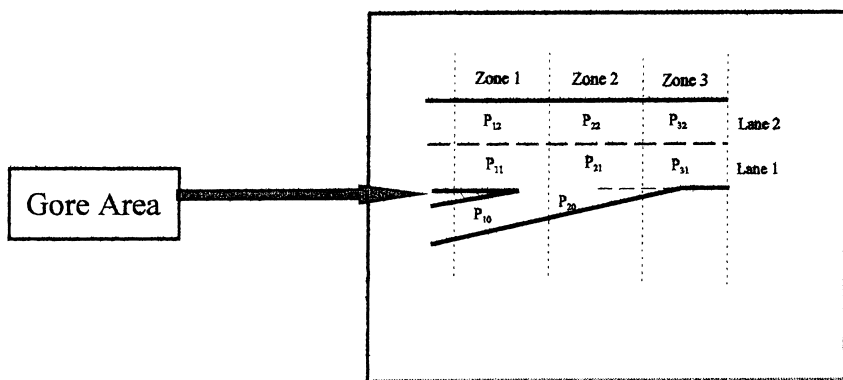


Fig. 2. Zonal probabilities.

where  $P_{10}$  and  $P_{20}$  are the probabilities that a merging vehicle is present in zones 1 and 2, respectively, and  $P_{11}$  through  $P_{32}$  are the probabilities of a vehicle in that zone and lane.

### 3.1. Model development

The model was developed through the following steps:

- (a) Identification of states,
- (b) Definition of breakdown,
- (c) Probability of transitioning from state to state,
- (d) Probability of breakdown occurrence.

#### 3.1.1. Identification of states

As discussed previously, there are eight zone–lane pairs, and hence there are  $2^8$  or 256 possible combinations of states. An example definition for a state (state 56) is presented in Fig. 3, where ‘1’ represents a vehicle present in that zone–lane pair and ‘0’ represents no vehicle present. These states were then utilized to develop a 256 by 256 matrix of the probability of transitioning from any one state to another. The time to transition from state to state was assumed to be 2 s based on 60-mph speeds and 175-ft zones.

#### 3.1.2. Definition of breakdown

For this paper, breakdown is defined as any time there is a merging vehicle present, and one of the following occurs for a minimum time span of 6 s (the time to transition three states total or two states after the initial state):

- (a) Vehicles are present in lane 1 and vehicles continually arrive in that lane by moving forward in lane 1 and/or merging from lane 2,
- (b) Vehicles are present in lane 1 and vehicles are braking in lane 1.

Breakdown is defined as described above since the state immediately following this situation would have experienced the inability to merge for at least 6 s. There were 47 initial states which met the above requirements that may potentially lead to breakdown. For example, state 68, shown in Fig. 4 is a potential breakdown state because the merge vehicle could possibly be forced

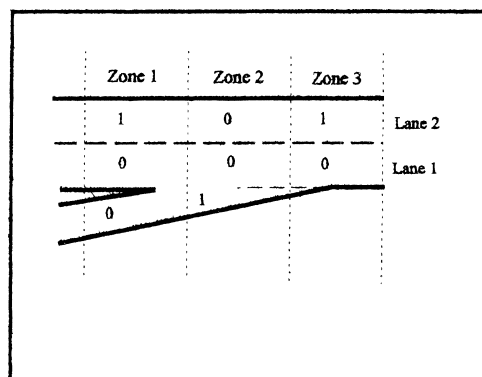


Fig. 3. Definition of state 56.

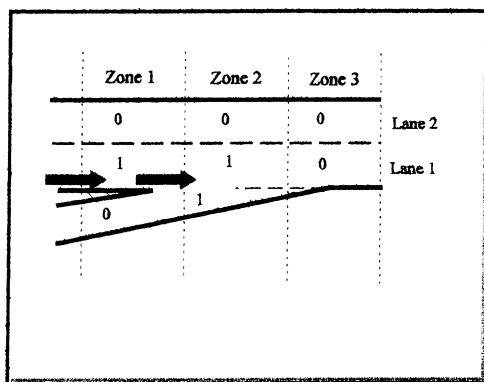


Fig. 4. Definition of state 68.

to brake for three consecutive states if vehicles continue to move forward into the zone–lane pair adjacent to it (as demonstrated by the arrows). Such states which are potential breakdown states are called ‘bad states’ and other states are called ‘good states’.

3.1.3. Probability of transitioning from any state to another state

To set up the 256 states in a matrix, the probability of transitioning from any one state to another state was determined. The transition probability is computed as the multiplication of the probabilities of vehicles arriving or not arriving, or being present (1) or not present (0), in each zone for each lane at the next state. It is assumed that a vehicle can only make one of three moves in transitioning from state to state. A vehicle can:

- (a) Brake and remain in its current position if a vehicle is present in its forward-moving and lane-changing path, or in the adjacent zone and lane for a merge vehicle;
- (b) Continue to move in its current lane to the next zone; or
- (c) Change lanes and move forward to the next zone (with the exception that vehicles will never move from freeway mainstream lanes to the merge lane).

With the above assumption, then, the general logic in determining  $P_{ij}$ , probability of transitioning from state  $i$  to state  $j$ , is to solve for the product of each individual probability of vehicles in each zone–lane pair at state  $i$  to transition, as necessary, to achieve state  $j$ . For example, state 1 is defined as the state with no vehicles present in any of the zone–lane pairs, shown in Fig. 5 and state 2 is defined as the state with only one vehicle present in zone 1–lane 2, shown in Fig. 6. To find the probability of transitioning from state 1 to state 2, the following formula is applied:

$$P(\text{state 1} \Rightarrow \text{state 2}) = (P_{n@12})(1 - P_{n@11})(1 - P_{n@10})(1)(1)(1)(1)(1), \tag{5}$$

where  $P_{n@12}$  is the probability of a new arrival in zone 1–lane 2,  $P_{n@11}$  the probability of a new arrival in zone 1–lane 1,  $P_{n@10}$  the probability of a new arrival in zone 1–merge lane and 1 is the probability of lane changing or moving forward into a zone–lane pair if no vehicle is present in this zone–lane pair.

In this example, the probability of a vehicle not being present in state 2 is 1 for all lanes in zones 2 and 3, since there were no vehicles present in zone 1, 2, or 3 in state 1 to possibly transition to zone 2 or 3. Values for the probabilities of braking, moving forward, and changing lanes were

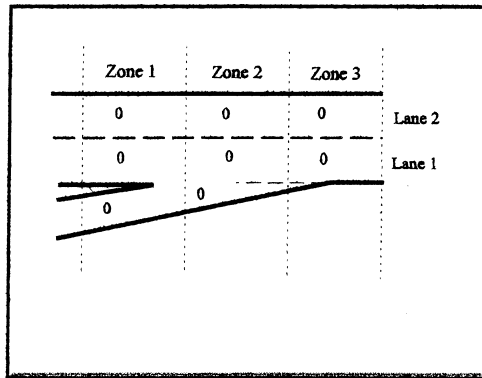


Fig. 5. Definition of state 1.

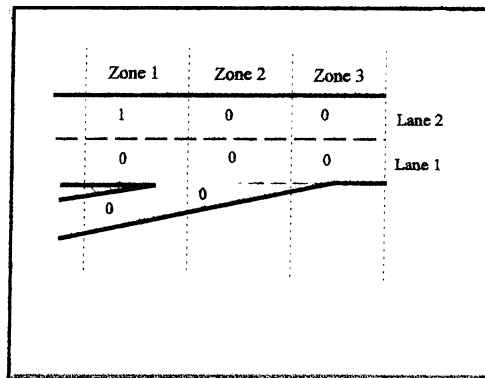


Fig. 6. Definition of state 2.

assumed based on the authors’ engineering judgement and were to be refined through a trial-and-error process. The twenty possible probabilities used in the equation for the probability of transitioning from state to state are listed and defined in Table 1.

Another example would be if in state  $i$  there were vehicles in zone–lane pairs zone 1–lane 2 (vehicle A), zone 2–lane 1 (vehicle B), and zone 2–lane 0 (vehicle C), and in state  $j$  there were vehicles in zone 2–lane 2, zone 3–lane 2, and zone 3–lane 1. In order to get from state  $i$  to state  $j$ , based on our previously listed assumptions, vehicle A had to move forward to zone 2–lane 2, vehicle B needed to change lanes to zone 3–lane 2, and vehicle C needed to change lanes to zone 3–lane 1. Hence, the probability of transitioning from state  $i$  to state  $j$  would be equal to the product of the probabilities of no new vehicles arriving, vehicle A moving forward from zone 1–lane 2 to zone 2–lane 2, vehicle B changing lanes from zone 2–lane 1 to zone 3–lane 2, and vehicle C changing lanes from zone 2–merge lane to zone 3–lane 1. This is demonstrated by the formula shown below:

$$P(\text{state } i \Rightarrow \text{state } j) = (1 - P_{n@12})(1 - P_{n@11})(1 - P_{n@10})(P_{mf@22})(P_{lc@32})(P_{lc@31})(1)(1), \quad (6)$$



Table 1  
Probabilities for transitioning from state to state

Probability	Definition
$P_{n@12}$	Probability of a new arrival at zone 1–lane 2
$P_{n@11}$	Probability of a new arrival at zone 1–lane 1
$P_{n@10}$	Probability of a new arrival at zone 1–lane 0
$P_{mf@22}$	Probability of moving forward to zone 2–lane 2
$P_{mf@21}$	Probability of moving forward to zone 2–lane 1
$P_{mf@20}$	Probability of moving forward to zone 2–lane 0
$P_{mf@32}$	Probability of moving forward to zone 3–lane 2
$P_{mf@31}$	Probability of moving forward to zone 3–lane 1
$P_{lc@22}$	Probability of lane changing to zone 2–lane 2
$P_{lc2@21}$	Probability of lane changing from lane 2 to zone 2–lane 1
$P_{lc0@21}$	Probability of lane changing from lane 0 to zone 2–lane 1
$P_{lc@32}$	Probability of lane changing to zone 3–lane 2
$P_{lc2@31}$	Probability of lane changing from lane 2 to zone 3–lane 1
$P_{lc0@31}$	Probability of lane changing from lane 0 to zone 3–lane 1
$P_{b@12}$	Probability of braking in zone 1–lane 2
$P_{b@11}$	Probability of braking in zone 1–lane 1
$P_{b@10}$	Probability of braking in zone 1–lane 0
$P_{b@22}$	Probability of braking in zone 2–lane 2
$P_{b@21}$	Probability of braking in zone 2–lane 1
$P_{b@20}$	Probability of braking in zone 2–lane 0

where  $P_{n@12}$  is the probability of a new arrival in zone 1–lane 2,  $P_{n@11}$  the probability of a new arrival in zone 1–lane 1,  $P_{n@10}$  the probability of a new arrival in zone 1–merge lane,  $P_{mf@22}$  the probability of moving forward to zone 2–lane 2,  $P_{lc@32}$  the probability of lane changing to zone 3–lane 2,  $P_{lc@31}$  the probability of lane changing to zone 3–lane 1 and 1 is the probability of lane changing or moving forward into zone 2–lane 1 or zone 2–merge lane.

If any state  $i$  cannot transition to another state  $j$  due to the assumptions of the three possible vehicle transitioning moves, then the probability of transitioning, from  $i$  to  $j$ , is zero. After the matrix was completed with equations for the probabilities of transitioning from  $i$  to  $j$ , the matrix was sorted by those states that will not lead to a breakdown two states later (‘Good’ states), and those states that may lead to a breakdown two states later (‘Bad’ states). A sample of the transition probability matrix is shown in Table 2. It should be noted that to check the matrix, all rows needed to sum to one.

### 3.1.4. Probability of breakdown occurrence

Utilizing the transition probability matrix and applying Markov Chains, the probability distribution of the time of breakdown occurring can be determined. This was completed using the defined assumption of 6-s time span as the definition of breakdown. The 6-s refers to the time to transition from any one state to two states later in time.

To develop the model a probability mass function/density function was developed as described below:

$P$ : is the probability transition matrix =  $[p_{ij}]$ ,

Table 2  
Sample transition probability matrix<sup>a</sup>

State	239	240	243	247	250	254	68	72	103	107	133	137	139	143	145
236	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
237	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
238	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
239	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
240	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
243	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
247	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
250	0.02	0.13	0.00	0.04	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
254	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
68	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.10</b>	<b>0.01</b>
72	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.10</b>	<b>0.00</b>	<b>0.07</b>	<b>0.00</b>	<b>0.02</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
103	<b>0.00</b>	<b>0.00</b>	<b>0.01</b>	<b>0.00</b>	<b>0.01</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
107	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.14</b>	<b>0.00</b>	<b>0.00</b>
133	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
137	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.05</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
139	<b>0.03</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
143	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
145	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.04</b>	<b>0.03</b>
146	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.04</b>	<b>0.03</b>
168	<b>0.00</b>	<b>0.00</b>	<b>0.05</b>	<b>0.00</b>	<b>0.04</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
172	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
174	<b>0.00</b>	<b>0.20</b>	<b>0.00</b>	<b>0.06</b>	<b>0.00</b>	<b>0.04</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
178	<b>0.00</b>	<b>0.00</b>	<b>0.01</b>	<b>0.00</b>	<b>0.01</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
180	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
181	<b>0.00</b>	<b>0.00</b>	<b>0.01</b>	<b>0.00</b>	<b>0.01</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>

<sup>a</sup> Non-italics, non-bold – Good to Good transition; italics, non-bold – Good to Bad transition; non-italics, bold – Bad to Good transition; italics, bold – Bad to Bad transition.

S: is the state space of Markov chains.

The matrix was partitioned into state spaces *G* (‘Good’ states – those states that do not include the potential for a vehicle to brake) and *B* (‘Bad’ states – those states that include the potential for a vehicle to brake) as follows:

$$P = \begin{bmatrix} p_{GG} & p_{GB} \\ p_{BG} & p_{BB} \end{bmatrix}, \tag{7}$$

where  $P_{BB}$  is the transition matrix of going from a ‘Bad’ state to another ‘Bad’ state.

Define  $q_n(i) = P_{BD}$  occurs after  $n$ th time slot/ $X_0 = i$ , where  $BD$  is the breakdown ( $\equiv$  3 consecutive bad states) and  $X_0$  the initial state.

Therefore for  $n \geq 2$ ,

$$q_{n+1}(i) = \begin{cases} \sum_j q_n(j)p_{ij} & \text{if } i \in G, \\ \sum_k q_{n-1}(k)p_{jk} & \text{if } i \in B, j \in G, \\ \sum_l q_{n-2}(l)p_{kl} & \text{if } i \in B, j \in B, k \in G, \\ 0 & \text{if } i \in B, j \in B, k \in B. \end{cases} \tag{8}$$

On conditioning, we get

$$q_{n+1}(i) = \begin{cases} \sum_j q_n(j)p_{ij} & \text{if } i \in G, \\ \sum_{j \in G} \left( \sum_k q_{n-1}(k)p_{jk} \right) p_{ij} + \sum_{k \in G} \sum_{j \in G} \left( \sum_l q_{n-2}(l)p_{kl} \right) p_{ij} p_{jk} & \text{if } i \in B \end{cases} \tag{9}$$

with

$$q_0(i) = 1 \forall i, \tag{10}$$

$$q_1(i) = 1 \forall i, \tag{11}$$

$$q_2(i) = \begin{cases} 1 & \text{if } i \in G, \\ 1 - \sum_{j \in G} [p_B^2]_{ij} & \text{if } i \in B. \end{cases} \tag{12}$$

Then, define for  $n \geq 1$ , and all  $i$

$$\begin{aligned} f_n(i) &= P \{ \text{Breakdown occurs at } n\text{th time slot} / X_0 = i \} \\ &= q_{n-1}(i) - q_n(i). \end{aligned} \tag{13}$$

Therefore the distribution of the breakdown times can be plotted.

It should be noted that once three consecutive bad states occur, the model recognizes this as breakdown. The model will not continue to read each additional bad state as another separate breakdown. In other words, if five bad states would occur in a row, the model would recognize this as one breakdown, not three.

A sample of the partitioned transition probability matrix with the initial values for this model may be found in Table 2 shown earlier. The MATLAB<sup>®</sup> code used to define this model may be seen in the appendices of Jodie (O’Leath) Evans’ thesis (O’Leath, 1998). The model was run for varying flow rates by editing the partitioned matrix, and the cumulative probability of breakdown versus time graphical outputs is displayed in Figs. 7–9.

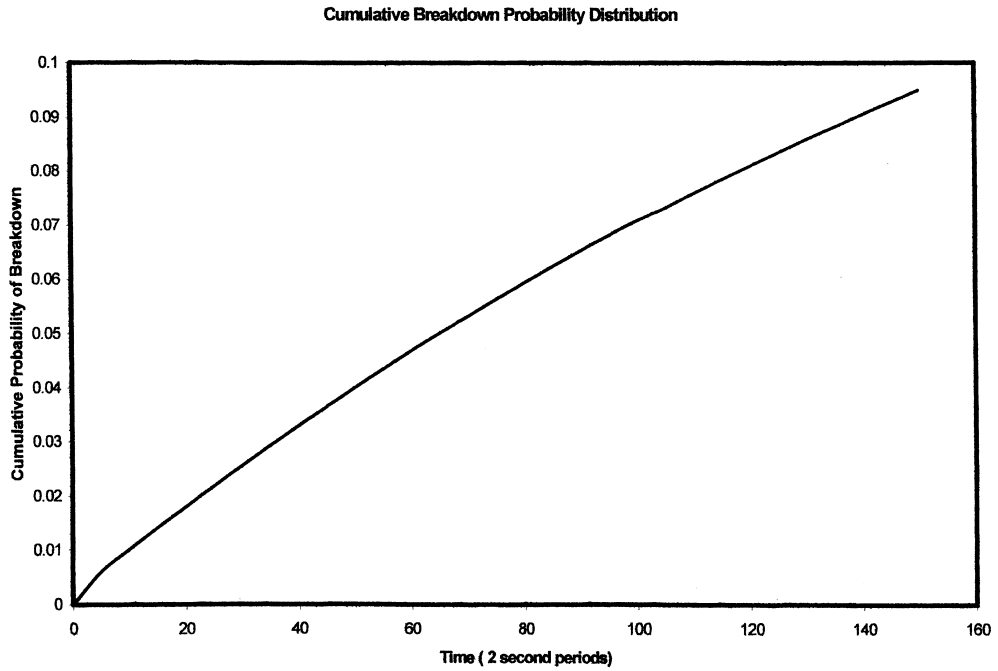


Fig. 7. Graphical output for (1500/1500/600 vph arrival rates for lanes 2/1/0 and  $n = 1:150$ ).

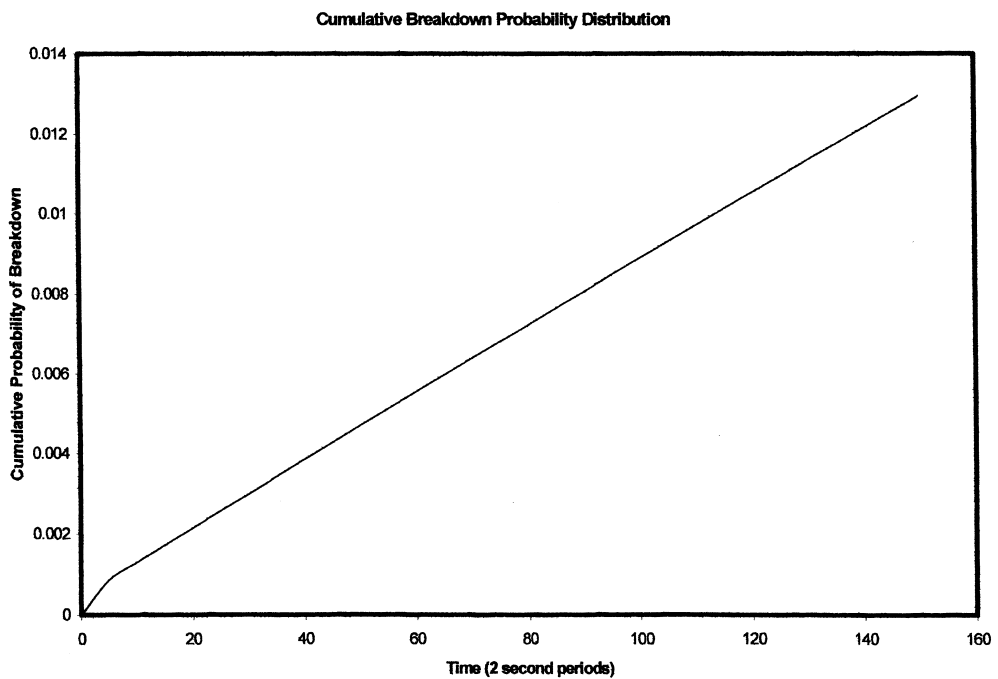


Fig. 8. Graphical output for (1000/1000/400 vph arrival rates for lanes 2/1/0 and  $n = 1:150$ ).

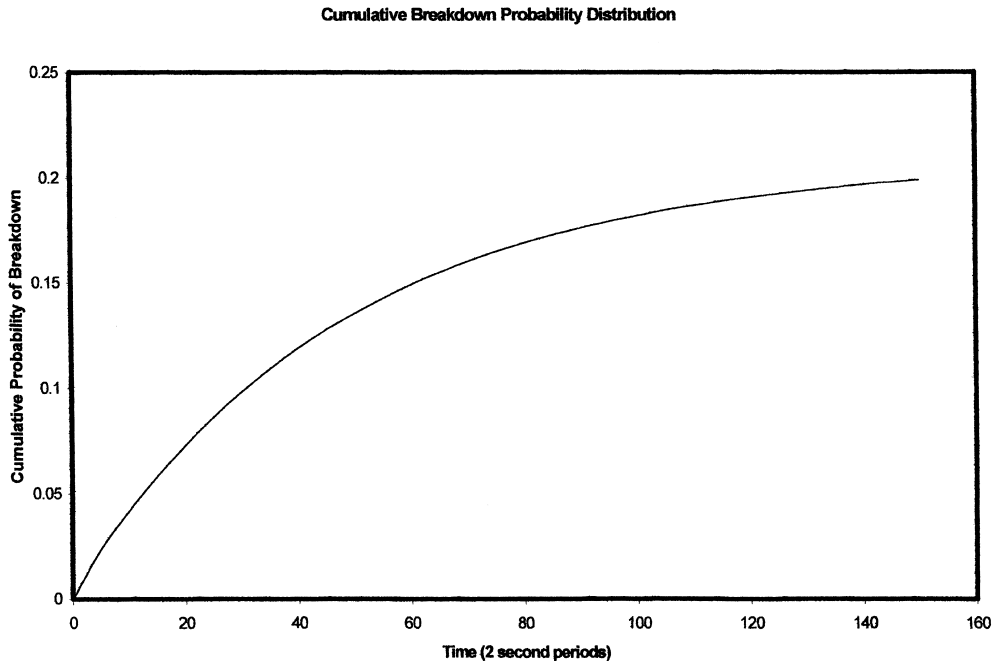


Fig. 9. Graphical output for (2000/2000/800 vph arrival rates for lanes 2/1/0 and  $n = 1:150$ ).

In explaining the graphical output in Figs. 7–9, a probability of breakdown of 0.01 signifies a 1% chance of breakdown occurring within the given relational number of 2-s time periods. For example, to find the probability distribution of the time of breakdown occurring within 5 min for a freeway with arrival rates of 1500/1500/600 vph for lanes 2/1/0 and  $n = 1:150$ , one would locate 150 2-s time periods (300 s/(2-s/time period)) along the  $x$ -axis of Fig. 7 and find the corresponding breakdown based on the graphical relationship. In this example, the probability distribution of the time of breakdown would be 0.09% or 9%.

In viewing the graphical output, it may be noticed that the curve only extends to 150 along the  $x$ -axis which represents time in 2-s periods. This is attributed to the application of the model with  $n = 150$  (where  $n$  is a 2-s time slot). Although a very large value of  $n$ , such as 3600, would be desirable, the model was applied with a limited value for  $n$  due to the extensive amount of time it would take to apply the model with a larger value in MATLAB<sup>®</sup> code. For example, about 16 h were required for the  $n = 150$  run, and nearly two days were required for  $n = 300$ . To produce an improved output from the model, a high-powered computer should be used with the application of  $n = 3600$ .

### 3.2. Data collection

Data were collected at a site in Chicago, Illinois. The data were collected using videotape at three different locations at the site. The three locations included one near the gore point of the merge, one near the ending taper of the merge lane, and one in the center between the previous two views. A diagram of the data collection locations is provided in Fig. 10.

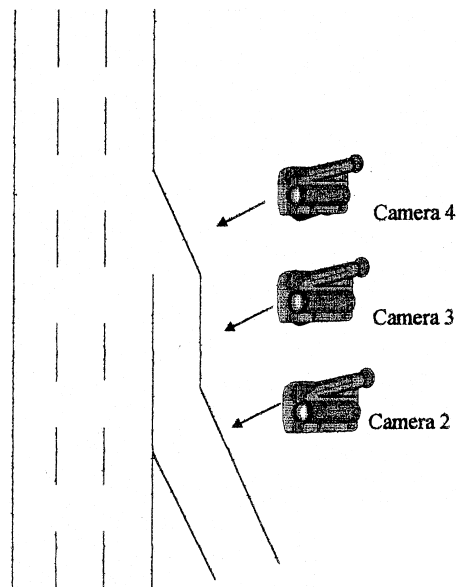


Fig. 10. Data collection locations.

### 3.3. Data analysis

For the data analysis the following tasks were completed;

- (a) Collected the arrival rates and presence in each zone–lane pair;
- (b) Determined the mean arrival rates and probabilities of presence; and
- (c) Inserted the arrival rates into the matrix.

#### 3.3.1. Arrival rates and presence in each zone–lane pair

Initially, a machine vision technology product was applied for the data reduction. This product can collect information from either a video camera or VCR tape based on user-specified detector locations.

When manual counts were conducted through viewing the videotapes, the results were found to be inaccurate. Therefore, it was decided that the manual counts would be used for model validation rather than the product's results.

#### 3.3.2. Mean probability of presence in each zone–lane pair

To validate the model, the probability of presence in each zone–lane pair was averaged for the site based on data collected. The manual data collection estimates were found by viewing the percentage of a given time interval (same as that used to determine the arrival rates) that a zone–lane pair had vehicles present. This could demonstrate whether the developed model can accurately determine the probability of presence in each zone–lane pair, and therefore indirectly validate the model.

### 3.3.3. Probability of transitioning from state to state

Once the arrival rates were determined, they were substituted in the probability equations in the matrix, along with the assumed values for braking, moving forward, and changing lanes. When the arrival rates as well as the probability of braking, moving forward, and changing lanes were substituted in the matrix for their respective variables, a numerical matrix was created allowing the application of Markov Chains for the determination of the probability distribution of the time of breakdown.

### 3.4. Comparison of probabilities of vehicle presence using the model with data

To determine the validity of the model, a comparison was made between the probabilities of vehicle presence (zonal probabilities) using the developed model and the values obtained through manual data collection. The model estimates were determined by finding the long run probabilities of vehicle presence in each zone–lane pair for the study site arrival rates. The manual data collection estimates were found by viewing the percentage of a given time interval (same as that used to determine the arrival rates) that a zone–lane pair had vehicles present. This was completed by counting the number of vehicles passing through each zone–lane pair by viewing the videotape, and another manual estimate was also attempted by taking snapshot views but did not produce as accurate results – most likely due to the fact that the snapshots were taken throughout the 2-h videotape, rather than being restricted to the same time interval as that used to find the arrival rates. The values for the model and data collection estimates are summarized in Table 3.

### 3.5. Model validity

After comparing the probabilities of vehicle presence using the developed model with the values obtained during data collection, it can be seen that the values are very close. Since the arrival rates were taken as vehicles entered zone 1 (zone–lane pairs 12, 11, and 10), it is logical that the model and data collection estimates would be nearly identical at this point.

It can be seen that the model estimation is fairly accurate as the vehicles move through zones 2 and 3. The probable reason for a slight reduction in accuracy is the fact that the lane-changing algorithm comes into application through zones 2 and 3. Therefore, it is expected that varying the transition probabilities in the lane-changing algorithm will probably remove the slight error.

Table 3  
Probability of presence for model and data

Zone–lane pair	Model estimate	Data collection estimate
12	0.44	0.44
11	0.31	0.31
10	0.14	0.13
22	0.48	0.47
21	0.32	0.35
20	0.19	0.11
32	0.49	0.46
31	0.38	0.42

Table 4  
Assumptions and their potential error in determining model validity

Assumption	Potential error
(1) Vehicles traveling at 60 mph with 2-s headways – allowing only one vehicle per 175-ft assumed zone	As breakdown potential increases the speeds drop below 60 mph and headways decrease to less than 2 s – allowing more than one vehicle per 175-ft zone
(2) Lane-changing, braking, and forward-moving probabilities based on engineering judgement	Without a validated algorithm of lane-changing, braking, and forward-moving behavior, it is difficult to justify the validity of the assumed values
(3) Model developed for four-lane freeway, and data collected from six-lane freeway sites	Additional lane for flow could effect lane-changing behavior, but since the number of vehicles lane-changing to and from the inside third lane was minimal, its effect is also expected to be minimal – also, the addition of three more cells would add tremendously to the number of states and, therefore, the research effort for little added value

Overall, the model appears to be valid with some restrictions due to several assumptions in the model. These assumptions and the errors they may have caused in validation are summarized in Table 4.

#### 4. Conclusions and future research

To develop an improved methodology for the prediction of breakdown, the probability distribution of the time of breakdown was determined based on the zonal merging probabilities with respect to the vehicles traveling on the throughway. Freeway flow, available gaps, and drivers' actions as they approach the merge area were taken into consideration in developing the model.

The analytical model was developed based on the information obtained from the literature and initial data sample. After determining the arrival distributions of the merging vehicles and the probability of transitioning from state to state, the probability distribution of the time of breakdown occurring was determined before any time,  $t$ , by applying Markov chains and implementing it in MATLAB<sup>®</sup> code. Finally, the data were reduced and applied in the model validation.

With respect to the application of Markov chains, it was expected that larger arrival rates of the vehicles traveling on the throughway would lead to higher probabilities of breakdown. This is logical since one would expect a higher probability of breakdown occurring when there are a large number of vehicles entering the system. The model results did predict a higher probability of breakdown for higher arrival rates.

The significance of the results are a better understanding of the prediction of breakdown and capacity, through the definition of breakdown, estimation. With an improved concept of breakdown on freeways, better freeway analysis and simulation software may be developed. Although the model may be improved to produce even more accurate results by applying smaller zone length and/or a more detailed lane-changing algorithm, it is important to note the logical nature of the model that was simply restricted by a few assumptions.



## References

- Akcelik, R., 1996. Relating flow, density, speed and travel time models for uninterrupted and interrupted traffic. *Traffic Engineering and Control* 37 (9), 511–516.
- Banks, J.H., 1991. Two-capacity phenomenon at freeway bottlenecks: a basis for ramp metering?. *Transportation Research Record* 1320, 83–90.
- Blumenfeld, D.E., Weiss, G.H., 1979. The effects of gap acceptance criteria on merging delay and capacity at an uncontrolled junction. *Traffic Engineering and Control* 20 (1), 16–20.
- Buhr, J.H., Drew, D.R., Wattleworth, J.A., Williams, T.G., 1967. A nationwide study of freeway merging operations. *HRR* 202, 76–122.
- Bunker, J.M., Troutbeck, R.J., 1996. The probability of delay to minor stream drivers at a limited priority freeway merge. *Road and Transport Research* 5 (1), 16–25.
- Cassidy, M.J., Windover, J.R., 1995. Methodology for assessing dynamics of freeway traffic flow. *Transportation Research Record* 1484, 73–79.
- Davis, G.A., Kang, J.G., 1993. Filtering and prediction on freeways using Markov models. *Pacific Rim Transportation Conference Proceedings* 1, 43–49.
- Drew, D.R., Buhr, J.H., Whitson, R.H., 1968. Determination of merging capacity and its applications to freeway design and control. *HRR* 244, 47–68.
- Elefteriadou, A., 1994. Probabilistic model of breakdown at freeway-merge junctions, Polytechnic University.
- Elefteriadou, L., Roess, R.P., McShane, W.R., 1995. Probabilistic nature of breakdown at freeway merge junctions. *Transportation Research Record* 1484, 80–89.
- Haight, F.A., Bisbee, E.F., Wojcik, C., 1962. Some mathematical aspects of the problem of merging. *Highway Research Board Bulletin* 356, 1–14.
- Hall, F.L., Agyemang-Duah, K., 1991. Freeway capacity drop and the definition of capacity. *Transportation Research Record* 1320, 91–98.
- Hiller, F.S., Lieberman, G.J., 1995. *Introduction to Operations Research*.
- Hunt, J.G., Yousif, S.Y., 1994. Traffic capacity at motorway roadworks – effects of layout, incidents and driver behaviour. In: *Proceedings of the Second International Symposium on Highway Capacity* 1, pp. 295–314.
- Lunefeld, H., 1993. Human factors associated with interchange design features. *Transportation Research Record* 1385, 84–89.
- Makigami, Y., Nakanishi, T., Toyama, M., Mizote, R., 1983. On a simulation model for the traffic stream in a freeway merging area. In: *Proceedings of the Eighth International Symposium on Transportation and Traffic Theory*, pp. 427–451.
- Michaels, R.M., Fazio, J., 1989. Driver behavior model of merging. *Transportation Research Record* 1213, 4–10.
- Nihan, N.L., Berg, D.B., 1992. A predictive algorithm for a real-time ramp control system. *ITE Journal* 62 (6), 29–32.
- O’Leath, J.L., 1998. Determination of the probability of breakdown on a freeway based on zonal merging probabilities. *The Pennsylvania State University*, 95.
- Olsen, R.A., Hostetter, R.S., 1976. Describing and shaping merging behavior of freeway drivers. *Transportation Research Record* 605, 7–13.
- Persaud, B., Yagar, S., Brownlee, R., 1998. Exploration of the breakdown phenomenon in freeway traffic. Preprint from TRB 77th Annual Meeting. 11–15 January.
- Szwed, N., Smith, N.M.H., 1972. Simulation of the merging process. *ARRB Conference Proceedings* 6 (3), 77–99.
- Yagar, S., Hui, R., 1996. Systemwide analysis of freeway improvements. *Transportation Research Record* 1554, 172–183.

## Further reading

- Barnes, K.E., 1993. Alternative method for analyzing merge/diverge and weaving areas on freeways with four or more directional lanes. Texas Transportation Institute.

- Chin, Quack, Cheu, 1991. Traffic conflicts in expressway merging. *Journal of Transportation Engineering* 117 (6), 633–643.
- Drew, D.R., Buhr, J.H., 1969. Gap acceptance and traffic interaction in the freeway merging process – phase II. Final report, Texas Transportation Institute.
- Hounsell, N.B., Barnard, S.R., McDonald, M., 1994. Capacity and flow breakdown on UK motorways. In: *Proceedings of the Second International Symposium on Highway Capacity 1*, pp. 277–293.
- Kulkarni, V.G., 1995. *Modeling and Analysis of Stochastic Systems*.
- Makigami, Y., Matsuo, 1990. A merging probability calculation method considering multiple merging phenomena. In: *Proceedings of the 11th International Symposium on Transportation and Traffic Theory*, pp. 21–38.
- Rappaport, S.S., 1981. Bounds on the probability of delay for queued vehicles waiting to merge. *IEEE Transactions on Vehicle Technology*, 182–186.
- Thompson, Young, 1988. Transtat: a microcomputer program for fitting probability distributions to traffic data. In: *14th ARRB Conference Proceedings Part 2*, 14, pp. 206–221.
- Tignor, 1975. Operational analysis of freeway moving-merge systems. *Transportation Research Record* 533, 1–21.
- Wattleworth, J.A., Buhr, J.H., Drew, D.R., Gerig Jr., F.A., 1967. Operational effects of some entrance ramp geometrics on freeway merging. *HRR* 208, 79–113.