

A Tunnel Freezing Process with Adaptive Control – A Stochastic Modeling Approach

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Abstract

The estimation of freezing times using various analytical and empirical methods have been implemented with limited success in the food processing industry. A critical process issue that needs to be considered is that of system control. Simulation models have indicated that a control policy where the conveyor speed is monitored and varied according to the current thermal load in the tunnel can provide significant energy savings. The primary objective of this paper is to provide a stochastic model for the tunnel freezing process with an adaptive control strategy.

Keywords: Continuous Time Markov Chain (CTMC), simulation-based control, process control

1.0 Introduction

For scenarios such as meat processing, where the thermal load is non-deterministic, a reliable estimation of freezing time requirements is critical for the operation and control of cryogenic systems. Traditionally, the operating parameters of a freezing tunnel are maintained at levels appropriate to freeze the maximum anticipated thermal load. This results in over-freezing of a significant percentage of the products. Moreover, the maximum thermal load cannot be predicted accurately for a process involving 'random' input. Therefore, 'under-freezing' of some products can occur. A similar problem does not exist for operations such as bottling, since the thermal load is deterministic [1].

Different thermodynamic models have been developed to describe the freezing process and provide methods to estimate the "dwell time" requirements for a product to be frozen. The two main thermodynamic models used to describe the freezing process are the 'heat conduction with temperature-dependent thermal properties model' and the 'unique phase-change front model' [2]. Most methods used to estimate dwell time requirements assume that the heat transfer in the freezing process occurs primarily due to conduction and convection. The most common approach is to model the heat transfer using Newton's law of cooling at the surface and to define an "effective heat transfer coefficient" to account for the net effect of all the actual heat transfer mechanisms involved. The difficulties in modeling the heat transfer process for irregular shaped objects necessitate the incorporation of several assumptions such as the existence of a steady state, uniform properties and / or shape approximations such as the object being treated as an infinite cylinder, sphere, or as a set of infinite parallel plates [3-9].

2.0 Problem Statement and Research Objective

Cryogenic freezing tunnels can be operated in a variety of modes. The need for alternative control strategies instead of the traditional industry practice of employing fixed operating parameters has been discussed in [1]. An 'intelligent' process control strategy which involved the continuous monitoring of product input and the controlling of either or both of the two primary control parameters - conveyor speed in the tunnel and the refrigerant flow was presented by the authors. The paper discussed the impact of varying one or both of the controllable variables adaptively for a known thermal mass distribution. Simulation models were used to obtain a preliminary understanding of the stochastic nature of the tunnel freezing process considered as a system and provide energy consumption comparisons.

For a normally distributed thermal load (thermal load $\sim N(\mu, \sigma)$) and a constant inter-arrival rate, the simulation results indicated that the freezer can be operated at a tonnage corresponding to the load of $\mu + 2.5\sigma$ instead of $\mu +$

4σ as is the practice in the traditional approach [1]. In this control policy, the entity, before entering the freezing tunnel checks the value of the current speed of the conveyor. If the required speed (function of its thermal mass and the current tonnage of the tunnel) for that entity is lower than that the current speed, the conveyor will be decelerated to a speed corresponding to the required time for that current entity. If the required speed is greater than the current speed, no change is made and the entity continues to move into the freezing tunnel. When an entity leaves the tunnel it verifies if it had been the most constraining entity. In such a scenario, the next most constraining entity's dwell time is used to set the speed of the conveyor. In the discussions in [1], the impact of different inter-arrival time and thermal mass distributions on this result has not been evaluated. A stochastic model for the tunnel freezing process using an adaptive control strategy is presented in this paper.

3.0 Stochastic Elements In Freezing Process

A schematic representation of the tunnel freezing operation is shown in Figure 1. The stochastic elements in the freezing process can be related to the product or to the process itself. The main product-related random variable is its thermal mass. Even though the thermal mass of each entity is a function of three other random variables – mass, thermal capacity (dependent on its composition), and the incoming temperature, it can be treated as a single random number. Process-related randomness is mainly in the form of the inter-arrival time distribution for the process [1].

3.1 Stochastic Model for Tunnel Freezing

Instead of trying to determine the optimal operating tonnage for a given thermal mass distribution and arrival pattern, as in the case of the simulation models in [1], it was decided to develop a model that will predict the “rejection” rate for a particular operating tonnage. The rejection rate refers to the percentage of the entities that will not be completely frozen in the first pass and hence will require additional pass (or passes) till the required level of freezing has been achieved. The model discussed here does not account for re-entrant entities. It assumes that the rework process is independent of the tunnel freezing process modeled here. In a subsequent paper, the impact of considering rework costs and re-entrant entities will be discussed. The tunnel freezing process has been modeled as a Continuous Time Markov Chain (CTMC). The elements of the CTMC will be discussed in the following sections.

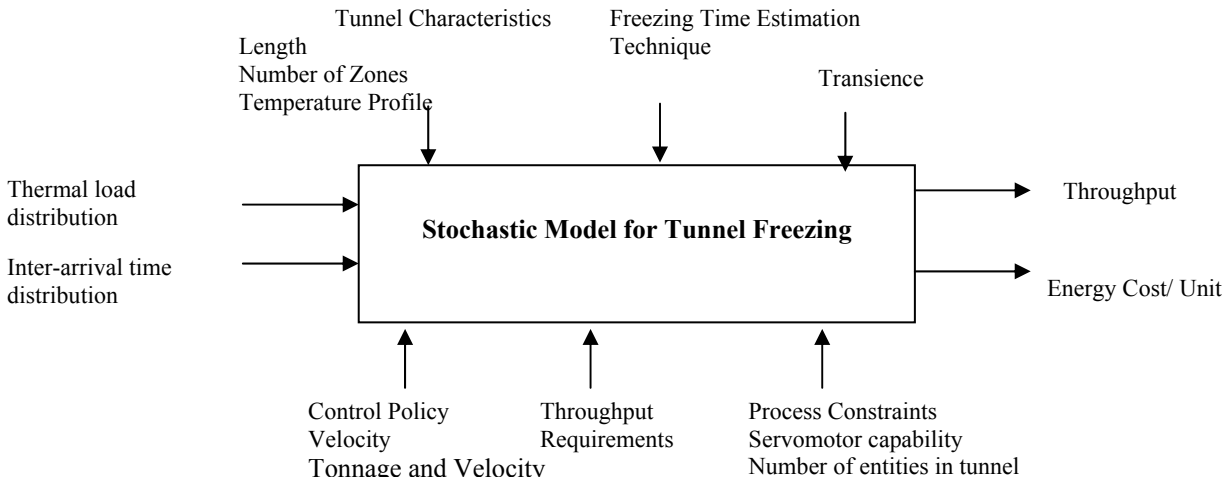


Figure 1. A schematic representation of tunnel freezing process as a stochastic model

3.2 Tunnel Freezing Process as a CTMC

CTMCs are systems which when enters a state k stays in that state for a random amount of time (sojourn time) that is exponentially distributed with parameter q_k ($q_k > 0$) and then jumps to a state l ($k \neq l$) with a probability p_{kl} [10]. The sojourn time and the new state depend only on the current state and given a state k , its sojourn time and the new state are independent of each other. The following sections discuss how the tunnel freezing process can be modeled as a CTMC.

3.2.1 Assumptions

This modeling approach assumes that the operating tonnage of the freezer for a particular ‘run’ is already known. Moreover, it is assumed that the range of thermal mass that will be exposed to the freezing process can be expressed as a discrete function. The thermal mass range is divided into $Class_{max}$ number of classes. The probability of an entity belonging to a class m is denoted as p_m . The velocity of the conveyor in the freezing tunnel is constrained by the entity with the maximum thermal mass currently in the tunnel. It will be assumed that the required velocity of any entity in a particular class is the same. This can be ensured by defining the velocities using the upper limit thermal mass in a particular class. Such an assumption, however, is bound to result in increased percentage of over-frozen entities.

The modeling and analysis also assumes that the process has already reached a steady state. It also assumes that the methodology is independent of the number of zones in the tunnel. Any influence of the tunnel characteristics is assumed to be captured in the parameter μ_j defined in the next section. Other issues such as the randomness associated with the size of the product and shape approximations for estimating dwell time requirements have not been considered in this model. It is also necessary to consider the time required for attaining steady state conditions and impact of “batch-changing” on the tunnel’s performance. Issues related to heat transfer in the tunnel during transient conditions is not considered in this model. Also, let V_j (for $1 \leq j \leq Class_{max}$) denote the velocity of the conveyor when the constraining entity is from class j (that means the entity with the maximum thermal mass currently on the conveyor, is from class j). V_j is therefore the conveyor velocity when there is at least one entity from class j and no entities from any classes ‘higher’ than j . Without loss of generality it is assumed that the larger the thermal mass, the higher the class. Let the maximum number of entities (from any subset of the different classes) that can be on the conveyor at any given time, t be denoted as $NTunnel_{max}$. The velocities V_j can be computed using the freezer tonnage and thermal capacity of class j . A Poisson arrival process with parameter λ [$PP(\lambda)$] is assumed for the process.

3.2.2 Model and State Space Definition

Let $X(t)$ denote the state of the system at time, t . The state of the CTMC has been defined using a 2-tuple (i, j) where:

i : the number of entities currently in the freezing tunnel ($1 \leq i \leq NTunnel_{max}$)

j : the class identification of the entity with the highest thermal mass, currently in the tunnel ($1 \leq j \leq Class_{max}$)

with the exception that state $(0,0)$ denotes no entities in the freezing tunnel. The rationale behind adopting the above 2-tuple is as follows. As discussed in a previous section, the velocity of the conveyor is dependent on the most constraining entity. It is necessary to store the class identification of the constraining entity in order to determine if any action is necessary when a new entity enters the tunnel or when an entity leaves the tunnel.

If the entering entity belongs to a class k such that $k \leq l$ then no change needs to be made to the conveyor. If the opposite holds true, the velocity has to be changed to suit the requirements of the new entity (the velocity of the conveyor will have to be decelerated. Notice that a new entity can only decrease the speed of the conveyor and never increase it). On the other hand, if the leaving entity belongs to the class to which the constraining entity belongs to (or it was the constraining itself), the velocity of the conveyor can be now increased. An entity leaving the tunnel cannot decrease the conveyor speed, but can only increase it. No change in velocity is observed when any of the following two situations occur (when the system is in state (i,j)):

- The entity leaving the tunnel does not belong to the class j
- The departing entity belongs to class j and there is at least one other entity belonging to class j still in the tunnel.

From the discussion above, it can be seen that the state space for the model considered is finite with the number of states being $NTunnel_{max} * Class_{max}$. As defined earlier, the arrival process is treated as $PP(\lambda)$. When the system is in state (i,j) the rate at which entities leave the tunnel is defined as μ_j where

$$\mu_j = \frac{V_j}{Length_Tunnel} \quad (1)$$

and $Length_Tunnel$ denotes the length of the tunnel freezer.

3.2.3 Generation of the Infinitesimal Generator Matrix of the CTMC

The next step is to determine the infinitesimal generator matrix for the CTMC. The number of states in this CTMC model is given by $N_{states} = NTunnel_{max} * Class_{max}$. Let (i,j) denote the current state of the system $(X(t))$, at time t . In order to illustrate the generation of the transition matrix for the discussed CTMC, a sample state space is shown in Figure 2. The two possible changes for the first component, i , in any one transition are:

- Decreases by 1 (departure of an entity). It can be seen that the rate at which this change could happen is $i\mu_j$, where the number of entities currently in the system is (i) and the current “service” rate for a single entity is μ_j . However, to determine the probability of transitioning into state $(i-1,k)$, an appropriate probability term must be multiplied to $i\mu_j$. This is discussed later. It is to be noted that when an entity leaves the system and happens to be from class j , the value of j can change or not depending on the absence or presence of at least one other entity belonging to the class j .
- Increases by 1, caused by an entity arrival. The value of j , in this case, can increase (to a value k , $j < k \leq Class_{max}$) or remain the same. The transition rates depend on the arrival process, which has been assumed to be $PP(\lambda)$. The complete transition functions will be explained next.

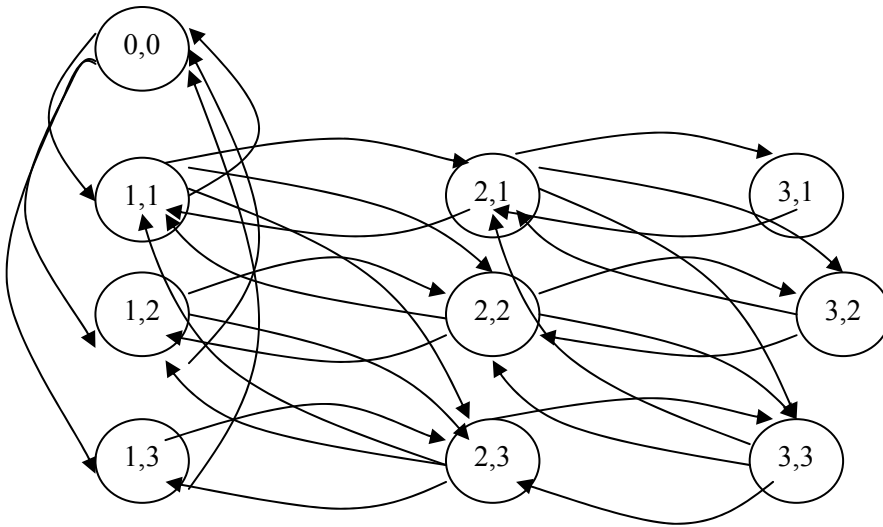


Figure 2. Representative state transitions in the CTMC model
(Assume $Class_{max} = 3$ and $Ntunnel_{max} = 3$)

3.2.3.1 Arrival Process

Let the state of the system $X(t)$ at time t be (i,j) . This section discusses the transition functions (i.e. corresponding rates) to all states of the form $(i+1, k)$. Based on the discussions in the earlier sections, the value of k cannot be less than the value of j . Let $q(i,j,l,k)$ denote the rate the CTMC transitions from state (i,j) to state (l,k) . When the value of k equals the value of j , the required transition rate is given as follows. This is dependent on the probability that the entering entity belongs to a class that is **not greater than** the current value of j .

$$q(i, j, i+1, k) = \lambda \sum_{x=1}^j p_x \quad (\text{For } k = j) \quad (2)$$

In scenarios where the value of k is in greater than j (and less than or equal to $Class_{max}$), the required transition function is dependent on the probability of an entity belonging to class k entering the system.

$$q(i, j, i+1, k) = \lambda p_k \quad (\forall k, j < k \leq Class_{max}) \quad (3)$$

When the value of k is equal to $Class_{max}$, the transition function reduces to λ from the above equation since the summation, $\sum_{x=1}^j p_x$ equals 1.

3.2.3.2 Departure Process

Now, consider the departure process. As before, let the state of the system $X(t)$ at time t be (i, j) . The transition functions that take the system to states of the form $(i-1, k)$ will be provided next. As discussed earlier, the value of k cannot be greater than the value of j . In order to represent the transition functions for the departure process, the following notation is introduced to represent the probability that an entity in the tunnel belongs to class i , given that all the entities are class j or lower.

$$r_i = \frac{p_i}{\sum_{m=1}^j p_m} \quad (4)$$

The required transition function can be then represented as follows

$$q(i, j, i-1, k) = i\mu_j \left[\left(\sum_{l=1}^k r_l \right)^{i-1} - \left(\sum_{l=1}^{k-1} r_l \right)^{i-1} \right] (\forall k, 1 < k \leq j) \quad (5)$$

When the value of $k = 1$, the required transition function is given as $i\mu_j p_1^{i-1}$.

It is to be noted that the sum of all the transitions must be equal to $i\mu_j$. For example, consider a system with 5 classes. Let the system $X(t)$ at time t be $(5, 4)$. We are interested in finding the transitions that lead to a state of the form $(4, k)$. The following table shows the associated transition functions.

From	To	Transition Function
$(5, 4)$.	$(4, 4)$.	$5\mu_4 [1 - (r_1 + r_2 + r_3)^4]$
	$(4, 3)$.	$5\mu_4 [(r_1 + r_2 + r_3)^4 - (r_1 + r_2)^4]$
	$(4, 2)$.	$5\mu_4 [(r_1 + r_2)^4 - (r_1)^4]$
	$(4, 1)$.	$5\mu_4 [p_1^4]$

Table 1. Example transition functions

3.2.4 Estimating Rejection Rate

As discussed earlier, one of the objectives of the CTMC modeling was to estimate the rejection rate of the system. The rejection rate can be obtained by considering the long-term probability of all states where an arriving entity will not be able to enter the tunnel. This corresponds to all the states of the form $(NTunnel_{max}, j)$. The rejection rate for the system can be obtained using the following equation

$$rejection_rate = \lambda * \sum_{i=1}^{Class\ max} \pi(NTunnel_{max}, i) \quad (6)$$

where

λ denotes the mean arrival rate (corresponding to $PP(\lambda)$)

$\pi(k, l)$ represents the steady state probability that the CTMC is in state (k, l) and can be computed using standard CTMC analysis [10].

4.0 Preliminary Results and Comparison to Simulation Results

A case study with the thermal masses normally distributed (truncated) was studied using the CTMC model, in order to compare its performance against the results indicated by the simulation model. The simulation model had indicated that the optimal operating tonnage for a rejection rate of 2% (not necessarily indicative of a typical freezing operation) was $\mu + 2.6\sigma$. This result is different from what was obtained in [1], because a $PP(\lambda)$ process was considered in this example, instead of the constant arrival rate assumption used in the earlier paper.

For the same conditions, the CTMC model's prediction of the optimal operating tonnage was seen to be dependent on the number of classes considered (as can be expected). As the number of classes increases, the number of discrete control points also increases, thereby increasing the efficiency of the system. With a class of 20, the optimal operating tonnage predicted by the CTMC model was $\mu + 2.8\sigma$. Only three different coefficient of correlations (five replications for the simulation model) were considered in the analysis (0.35, 0.50, 0.65). The results from the preliminary analysis show that the CTMC model is capable of predicting the optimal operating tonnage within a short range of the simulation results. A detailed analysis based a statistical design of experiments with the thermal load distribution, number of classes, and arrival rate as the main factors will be discussed in a subsequent paper.

5.0 Conclusions

An adaptive control (simulation-based) strategy for determining the conveyor velocity of the tunnel depending on the thermal mass of the entities (maximum thermal mass) was presented in an earlier paper. Here, an approach to model the tunnel freezing process as a CTMC was presented. This model can be used to predict the rejection rate at any operating tonnage when an adaptive control strategy is adopted. The optimal operating tonnage for a given thermal mass distribution can be determined by iteratively using the model until an acceptable rejection rate is found. This model does not constrain the thermal masses to be normally distributed. The ability to treat the thermal masses as a discrete function adds to the flexibility of the model. The modeling of the process as a CTMC enables determining optimal operating conditions for the freezing tunnel in less time compared to an approach based on simulation. However, the modeling approach cannot be used as a high level controller for active control purposes. A CTMC model that considers the impact of rework costs and re-entrant entities, and a methodology to map the results from the simulation model [1] and the CTMC model presented here will be discussed in a subsequent paper.

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